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THE STRENGTH OF THE EARTH'S CRUST

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PART V. THE DEPTH OF MASSES PRODUCING GRAVITY ANOMALIES AND DEFLECTION RESIDUALS

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INTRODUCTION AND SUMMARY

The fact that the observed deflections of the vertical are on the average only one-tenth as large as the computed effects of the topographic relief, computed on the assumption of uniform density throughout each earth shell, shows that the densities of the crust

[1] Section B of Part V, on the applications of the criteria to determine the limiting depths, forms, and masses of the excesses and defects of density, will be published in the following number of this *Journal*.

are not uniform, but in a broad way are balanced against the relief. This is the proof that a condition prevails of approximate regional isostasy. As the relief of the globe is highly variable, the densities in the lithosphere are therefore within certain limits also highly variable. But on the other hand, the existence of gravity anomalies and deflection residuals indicates that the variations in density are not completely in accord with the demands of the hypothesis which postulates local compensation of the topography uniformly distributed to a uniform depth, nor apparently with any other simple hypothesis. These quantities measure the differences between the hypothesis and the facts of nature. Let the density variations beyond those required to balance the topography vertically above them be called the outstanding excesses or defects of density and the masses which they represent be called the outstanding masses.

It is fundamental to the problems of the strength of the crust, and a system of geologic dynamics in accord with that strength, to determine the depth, form, and weight of these outstanding masses. Do they belong to the centrosphere or to the lithosphere? As Gilbert has noted, if they are to be referred to the centrosphere they do not imply any imperfection of isostasy nor any competence for stress within the crust. Or, if they exist in the lithosphere, the zone of compensation, but are balanced vertically in the same column by other masses of opposite sign, this arrangement will produce local strains within the crust but not tend to flex the crust as a whole. Neither in this case, therefore, would they measure departures from perfect isostasy. As following questions, are the imperfections of isostasy small and local, and the residuals and anomalies the summation of many scattered effects? Or, on the contrary, are there notable regional departures from the conditions of solid flotation which measure a very appreciable rigidity of the crust? If so, to what extent are these regional outstanding masses related to the mountains, valleys, and deltas in process of evolution under the present cycle of surface activities; producing a progressive unbalancing possibly being slowly restored toward balance by a viscous undertow? To what extent are the departures from isostasy due to variable composition and density of igneous intrusions dating back to earlier geologic ages, perhaps never in

isostatic balance, and supported permanently and rigidly by the strength of the crust?

These problems have been studied in the last three parts of this article by means of the evidence presented by Hayford and Bowie, but that evidence has not been used to solve the depth of the outstanding masses. Yet it is seen that all of the aspects just enumerated are bound up in that factor. It is especially this problem of the depth and the consequent areal extent and mass of the units which produce the residuals of the deflection and gravity observations which is attacked in this part. It is necessary for this investigation to enter into a study of the complex relations between the anomalies and residuals which depend upon the depth and form of masses. It is a subject upon which, so far as the writer is aware, but little has been done, so that about half of this chapter, published as Section A, consists of a study of these relations preliminary to their application.

For facility of mathematical treatment the individual outstanding masses must be regarded as equivalent to spheres, spheroids, or cylindrical disks, either as units or as aggregates. If only the epicenter (the point on the surface vertically above the center) of the disturbing mass is determinable and the deflections at two or three points on one side of it, then the mass may be most simply interpreted as a sphere; since the mass of a sphere acts as if concentrated at its center. With fuller observations a close approximation to the depth of the mass and a less close approximation to its form and density may be made. The first problem then is to determine the epicenter of the outstanding mass and its depth, using for this purpose the nature of the anomalies and residuals. For a sphere beneath a plane surface it is shown that the value of the maximum gravity anomaly at the surface is 2.6 times the value of the maximum deflection residual, both being measured in the same units of force. The former occurs vertically over the abnormal spherical mass, that is, at the epicenter. The latter occurs at a horizontal distance from the epicenter equal to 70 per cent of the vertical depth to the center of mass. Oblate spheroids and broad cylindrical disks with vertical axes and the same depth of center as the sphere give maximum deflections at greater

distances from the epicenter. The curves of the deflection force for these forms, especially the spheroidal forms, resemble somewhat closely those given by deeper spheres of greater mass. If the outstanding masses are in reality horizontally extended the interpretation of the deflection residuals as due to spherical masses assigns to their centers in consequence too great a depth. If, however, the masses have the forms of vertical prolate spheroids or vertical elongate cylinders, the interpretation as spheres will give too shallow a depth. The ratios of the maximum anomalies to the maximum deflections constitute a criterion to show whether the masses depart from spheres by the spreading-out of their substance in a horizontal plane or along a vertical axis.

In Section B the outstanding masses are shown in some cases to be horizontally extended in form and this is thought for the larger masses to be a rather general relationship; that is, the vertical thickness is much less than the length or breadth. Consequently the interpretation as spheres gives maximum limits to the depth.

A general inspection of the geodetic data as well as a detailed study of a certain test region shows that the smaller disturbing masses have their centers in the outer third of the zone of compensation; that is, within 40 km. of the surface. This result is to be expected, since similar small masses at greater depth would not exert a notable effect because their gravitative force varies inversely with the square of the distance. But evidence of more significance is found in regard to the larger centers of outstanding mass not related to topography. These also are found, in so far as they have been investigated, to be situated in the outer third of the zone of compensation. Yet these masses are capable of showing notable effects to distances of from 100 to 150 km. If they were situated at any depth within the zone of compensation they would, therefore, betray both their existence and their depth. The greater departures from isostasy appear, therefore, to be really absent from the deeper parts of the lithosphere.

Centrospheric heterogeneity, if present, would require greater masses in order to show surface effects. But no such effects are noted. In so far as they may be existent, they are largely masked

by the more important attractions of superficial masses and hidden by the indeterminate nature of much of the present data. Therefore centrospheric heterogeneity is not a hypothesis which can be used to account for the apparent departures from isostasy. It can be at most only a very secondary factor.

In the last topic of Section B is discussed the relation of the depth of outstanding masses to the various hypotheses regarding the distribution of compensation. The hypothesis of local compensation, as is perceived to a certain extent by those who have used it, is in error in supposing that variations in density correspond to every topographic feature and extend uniformly to the bottom of the zone. But these errors, whether they be small or great, are so spread out in depth and their centers of attraction are consequently so far removed from the surface that they have little effect on the geodetic observations. Especially is this true in comparison with those large and concentrated outstanding masses due to batholithic invasion or other causes which are found to exist at moderate depths in the outer crust. The reasons then why the deflection residuals and gravity anomalies appear to show so little relation to local surface relief and larger physiographic provinces are threefold: in part because of a regional compensation; in part because the hypothesis of local compensation as here shown masks the error contained in the assumption of perfect and local isostasy; in part because for many regions the ancient heterogeneities of mass hidden within the crust seem in reality to be greater than the heterogeneities of mass visible at the surface in topographic form and created by present gradational actions.

The results of this chapter converge with the lines of evidence previously considered and confirm them in showing considerable defects from isostasy for areas which are 100 km. or more in radius. This confirmation is to be expected, since it would be indeed remarkable if a crust, competent to carry such loads as the *geologic* evidence from erosion and sedimentation shows to be imposed, should give *geodetic* evidence of fairly local and nearly perfect adjustment between the topographic forms, developed by present external processes, and the variations in density imposed by past internal forces.

SECTION A

DEVELOPMENT OF CRITERIA FOR SPHEROIDAL MASSES

Separation of lithospheric from centrospheric outstanding masses.—Let the zone of compensation be regarded as the boundary of the lithosphere. At its bottom consider to exist a zone in which that lateral flowage takes place which is necessary for movements of isostatic readjustment and the maintenance through geologic time of a condition of more or less complete isostasy. Below it is the inner and more rigid core of the earth, the centrosphere. Let those excesses or defects of density above the zone of isostatic flow which are not in accord with the isostatic compensation of the topography be designated for convenience as lithospheric outstanding masses. Let all heterogeneities of density within any earth shell below the zone of isostatic flow be called centrospheric outstanding masses.

In his recent paper on the "Interpretation of Anomalies of Gravity,"[1] Gilbert calls attention to the fact that if abnormalities of density exist below the zone of compensation they will produce anomalies of gravity without these signifying real departures from isostasy. This is a very necessary addition to the theory of the cause of gravity anomalies and deflection residuals. As a test, Gilbert has calculated the influence of a right cylinder with vertical axis, of density ± 0.025, with height and radius each equal to 122 km., whose upper surface is at a depth of 122 km., thus reaching up to the bottom of the zone of compensation as given by Solution H. Such a cylinder would give a maximum anomaly of ± 0.023 dyne at the epicenter, a quantity of the same order of magnitude as the mean anomaly for the United States, 0.018 or 0.020 dyne.

In the application of this test to the earth it would appear, however, that two things should be noted. First, to account for the *mean* anomaly of 0.020 dyne the centrospheric masses would have to be several times as great as this cylinder, even for this depth of 122 km. to the top surface, since the maximum value of the anomaly occurs at the epicenter of the mass, and for a cylinder of

[1] *Professional Paper 85C, U.S. Geol. Survey*, 1913, pp. 35, 36.

the form postulated falls off rapidly with increasing horizontal distance from the epicenter. Second, the test mass has been taken as contiguous to the zone of compensation above and with that limited depth given by the hypothesis of uniform compensation. This gives it greater effect according to the law of inverse squares, but postulates either an indefinitely thin zone of isostatic flow at the bottom of the zone of compensation or a capacity in a thicker zone of weakness to maintain within itself heterogeneities of density similar to those of the lithosphere above and the centrosphere beneath. It is thought by the present writer that a more probable presumption is that the centrospheric heterogeneities which may exist are distinctly deeper than 122 km. and separated from the lithospheric outstanding masses by a thick zone which yields to broad inequalities of pressure either upon it or within it and therefore is incapable of maintaining notable inequalities of mass in this shell.

The reasons for this preliminary hypothesis are briefly as follows: The depth of compensation seems to be variable and to extend in some regions to as much as 300 km., even under the assumption of compensation uniformly distributed and complete at the bottom. Under a more natural assumption that isostatic compensation gradually disappears, those heterogeneities of density which give isostatic compensation would gradually diminish with depth and this diminution would extend to a considerably greater depth than 122 km. If heterogeneities which act isostatically gradually disappear, the heterogeneities which can be borne in excess should also be expected to diminish.

As to the nature of the shell immediately below the zone of compensation, Schweydar has recently analyzed mathematically the results of the measurements of earth tides by means of the horizontal pendulum.

The calculations were designed to test the presence or absence of a viscous zone between an elastic crust and elastic interior. It is concluded that even a magma bed with a viscosity as high as that of sealing wax at house temperatures and a thickness of but 100 km. cannot be present. The assumption in best agreement

with observations is that of the presence of a layer about 600 km. thick, slightly ductile (coefficient 10^{13} to 10^{14}), existing beneath an outer crust 120 km. thick.[1]

By postulating such a thick zone for isostatic flow, the viscous resistances are reduced and solid flow is made easier. It also is in conformity with the probability of a gradual change of physical state from the rigidity above into a less rigid and less stable tract and this in turn into a more rigid interior. Now if such a thick viscous zone is incapable of supporting over broad areas loads imposed by abnormalities of density above, it should also be incapable of supporting such horizontal inequalities of mass within it, provided these are sufficiently large. But in order to produce the same gravitative surface effects as more superficial masses, the heterogeneities of this zone of viscous flow would in fact have to be much larger. A cylinder of the dimensions postulated by Gilbert, if of negative density and adjacent to another at the same depth but of positive departure from the mean density, would tend to be underthrust by the latter, and the denser would in turn tend to be overflowed by the lighter.

For these reasons it is not to be expected that the same departures from those densities giving isostatic equilibrium which could exist in a rigid shell above would extend immediately below.

Influence of centrospheric heterogeneity.—To test the question of the influence of heterogeneity below the zone of compensation a sphere will be considered. First, one whose center is at a depth of 319 km., 0.05 of the radius of the earth. As a second test, the influence will be determined of a sphere whose center is at a depth of 637 km., 0.10 of the earth's radius. For considering the attraction of a mass at points on the surface other than at the epicenter it is more convenient to take the mass as having the form of a sphere rather than a cylinder, since the mass of the sphere acts in all directions as if concentrated at its center. This favors, furthermore, the accentuation of the effects upon the surface over what they would be if the outstanding mass had a stratiform extension.

[1] Dr. Wilhelm Schweydar, "Untersuchungen über die Gezeiten der festen Erde und die hypothetische Magmaschicht," *Veröffentlichung des k.k. Preusz. geodät. Institutes*, Neue Folge No. 54, Leipzig, 1912 (B. G. Teubner).

The sphere having the same volume as a cylinder 122 km. in radius and 122 km. in depth will have a radius of 111 km. Let a radius of 100 km. and a density of ±0.025 be assumed as the dimensions and mass of a standard sphere in this deep zone. If the center of such a sphere is at a depth of 183 km., the same depth as the center of Gilbert's postulated cylinder, the anomaly at the epicenter will be 0.021 dyne, whereas the cylinder gave an anomaly of 0.023 dyne. They are therefore nearly equal in effect. If the center of the sphere is placed at a depth of 319 km., making the top at 219 km., the anomaly at the epicenter becomes 0.0068 dyne. Consequently the variation in density or volume of the sphere would have to become three times as great in order that its maximum anomaly should equal the mean observed anomaly. But as the average anomaly is not measured at the epicenter, and the maximum anomalies, occurring at the epicenters, are several times the observed mean anomalies, this figure would have to be still further multiplied. To account, therefore, for the magnitude of surface anomalies, the disturbing spheres, if with centers at a depth of 319 km. and if of 100 km. radius, would have to have abnormalities of densities ranging up to 0.25 in order of magnitude. If the centers of the spheres were at twice this depth the abnormalities, to produce the same effect, would have to be four times as great in mass. In a region of which there is no precise knowledge such variations of density might well occur. The problem must therefore be investigated by means of the gradients which would result in the gravity anomalies and deflection residuals and a comparison of these with the gradients actually observed and plotted.

Distribution of surface forces for centrospheric spheres.—For masses as deep as these the curvature of the earth becomes of importance, but the complications which it introduces into the analytic treatment have been avoided by means of a graphic solution.

In Fig. 8A, the anomaly is calculated for the epicenter. Then the gravitative force at any other point on the surface, such as that having a dip angle θ, can be determined by squaring the inverse ratio of distance. Multiplying the force at the epicenter by this

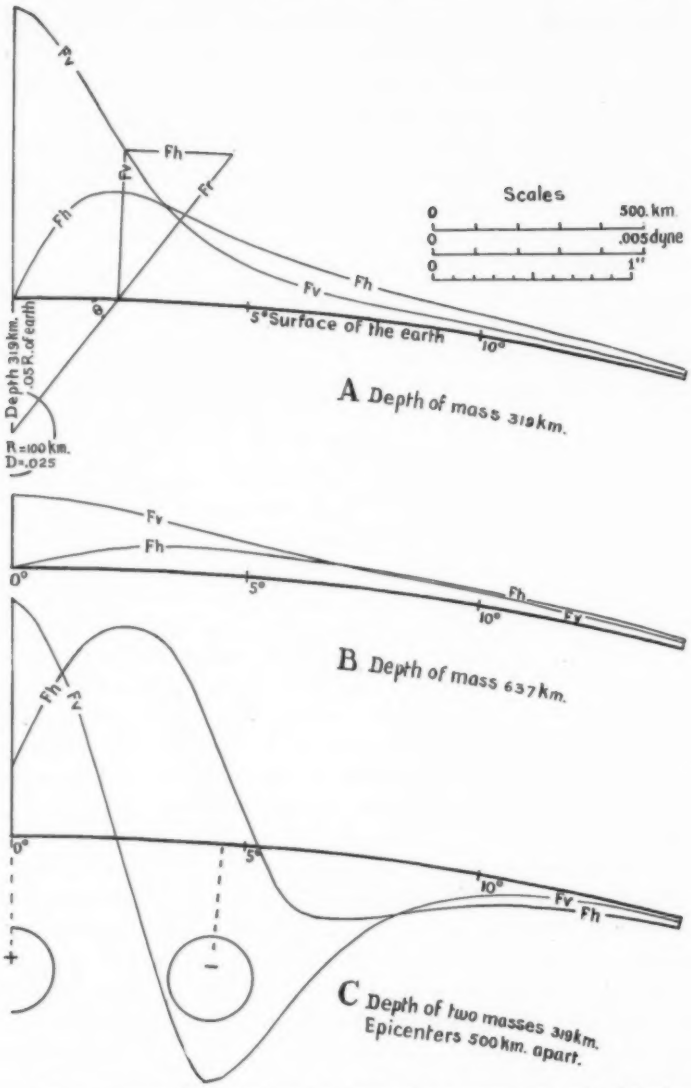

FIG. 8.—Curves showing values for the gravity anomalies *Fv*, and deflection residuals *Fh*, on the surface of the earth, for spherical masses situated at depths of 319 and 637 km., respectively. Ordinates measured at right angles to earth's surface.

factor gives the force at the second point acting in the direction of the radius of the attracting sphere. This value is laid off as *Fr* and then resolved into two components *Fv* and *Fh*, vertical and parallel respectively to the surface of the earth. The ratio of *Fv* to *Fh* is tan θ. With increasing distance from the epicenter θ becomes increasingly greater than it would be if the earth's surface were regarded as a plane. Therefore for distances of 5 to 10 degrees and more from the epicenter *Fv* begins to hold an appreciably higher ratio over *Fh* than it would if curvature were neglected. It is seen that $Fv=Fh$ for $\theta=45°$. Nearer the epicenter *Fv* is in excess; at greater distances *Fh* is the greater. *Fv* is a maximum for $\theta=90°$. *Fh* is a maximum for $\theta=55°$ if curvature be neglected. For the earth's curvature and a depth of 319 km. to the center of mass, θ is a maximum for $53°\pm$. The point giving this is at a distance from the epicenter of 0.75 the depth. The ratio of maximum *Fv* divided by maximum *Fh* is approximately 2.7. In Fig. 8C are shown the effects of two spheres of opposite sign but of equal mass. If these two spheres were superposed they would of course completely neutralize each other. Upon moving them horizontally apart to 1.5 times the depth, the maximum value of *Fh* becomes twice the value for a single sphere. This occurs halfway between them, and the value of *Fv* for this point is zero. The ratio of maximum *Fv* over maximum *Fh* becomes 1.1. Two equal masses of like sign would, on the contrary, give a maximum value of *Fv* and a zero value of *Fh* at a point halfway between them. These represent the extreme departures from the case of a single spherical disturbing mass. More distant masses show less overlapping of their fields of force and tend to have their individual effect upon a point between them neutralized by the larger number of masses acting from various directions. The values of *Fv* are much more under the control of the individual masses than are the values of *Fh*.

Returning to the single dominating mass of spherical form as shown in Fig. 8A, let the values of *Fv* and *Fh* be represented by ordinates as shown in the figure; then the surface representing the gravity anomalies, *Fv*, would be a dome of double curvature, like a craterless volcano; the surface representing the deflection

force, Fh, would be in the form of a caldera or crater ring. According as the attracting mass departed in its nature from a sphere these shapes of the force surfaces would be modified, but the general character of volcanic cone and caldera would remain. Negative masses would have the forms reversed. If the disturbing masses are at distances apart which average several times their depth, then a relief map of the resultant forces would resemble a volcanic field with the volcanoes isolated from each other. If the centers are much closer, the relief map would come more to resemble those lunar craters which show all degrees of superposition upon older craters. The deeper the masses the broader the volcano-like curves of forces upon the surface, but the lower will be the relief, unless the disturbing spheres increase in mass with the square of the depth. Stratiform-like masses, such as oblate spheroids or cylindrical disks, will show less pronounced effects than the equivalent spheres and will simulate somewhat the effects of spheres at a greater depth. The attempt to apply these principles as criteria to the published data must be deferred until the influence of abnormal masses in the zone of compensation has been considered and also in somewhat more detail the influence of masses in other forms than spheres.

Influence of spheres within the zone of compensation.—The forces produced by spheres within this zone are more readily treated analytically, since it will be seen that the curvature of the earth may be neglected. Otherwise this topic is to a considerable degree an extension of the last. The unit mass which it is convenient to adopt for this discussion is that of a sphere whose radius is 50 km. and density 0.100, one-half the mass of the sphere previously considered. Its center is taken at a depth of 64 km., 0.01 of the earth's radius, and approximately at the middle of the zone of compensation as given by Solution H. This gives the greatest abnormality of mass in the middle of the zone of compensation and will approximate to the mean effect of an outstanding density distributed uniformly throughout that zone. At the epicenter the attraction is wholly effective in producing gravity anomaly and is measured by the formula

$$F=\frac{dc(\frac{4}{3}\pi R^3)}{D^2}$$

In this d=density, c=constant of gravitation, R=radius, D=depth. Solving this equation for the values chosen gives

$$F = .0853 \text{ dyne}$$

Take the earth's surface as a plane and any point on it as located by the dip angle θ, made by a line from the point to the center of the sphere of outstanding mass. Then the vertical component Fv and the horizontal component Fh are given by the following equations:

$$Fv \text{ (dynes)} = 0.0853 \sin^3 \theta$$
$$Fh \text{ (dynes)} = 0.0853 \sin^2 \theta \cos \theta$$

To convert Fh into seconds of arc divide by 0.00475 and

$$Fh \text{ (seconds)} = 17.94 \sin^2 \theta \cos \theta$$

The maximum value of Fh occurs for $\theta = 55°$ and is 0.0328 dyne or 6.9 seconds. The curves for Fv and Fh are shown in the unbroken lines of Figs. 9 and 10. They are seen to be quite close in character to the curves of Fig. 8. Changes in the mass or depth of the sphere will serve to change only the scales of forces and distances so that these curves may be adapted readily to apply to all spherical masses situated within the lithosphere.

Influence of sum of intersecting spheres approximately equivalent to spheroids.—The analysis of the gravitative forces which a sphere exerts upon points in an external plane serves as a starting-point for the consideration of the problem of the influence of those unit masses of excess or defect of density which exist in the crust. As a further step, any one mass may be considered as approximating in form either to some oblate or prolate spheroid or to some ellipsoid of three unequal axes. But the equations for the forces exerted by spheroids upon an external plane are complicated and laborious to solve. A sufficient approximation to the influence of a spheroid may be made, however, by employing several intersecting spheres which together give an approximation to the right quantity and distribution of mass. The influence of the composite mass is readily attained by summing up the curves given by the modifications for the several spheres.

In Fig. 9 the unbroken lines, as previously noted, are the curves of force due to the single unit sphere. The broken lines show the

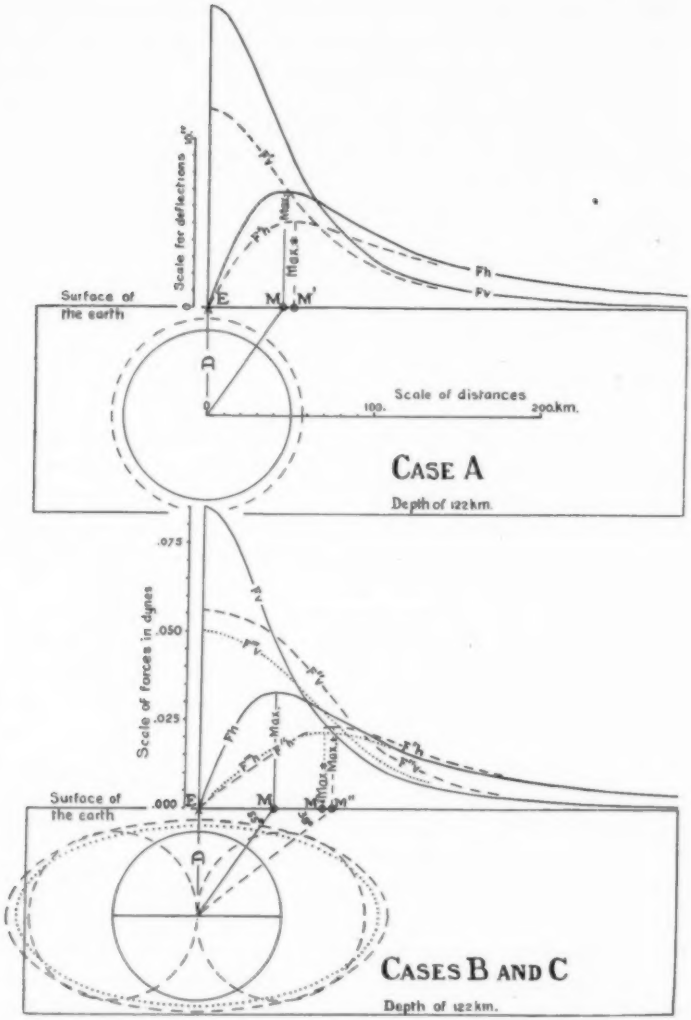

FIG. 9.—Various cases of horizontal and vertical components of the gravitative force due to a unit mass concentrated into a single sphere and expanded into several intersecting spheres. Cases A and B, three spheres in line. Case C, five spheres made by combining A and B and omitting one central sphere.

curves of force due to the same unit mass expanded into three spheres of the original dimensions and with centers 50 km. apart on a horizontal line. One-third of the mass is therefore in each sphere and the density of each is 0.033. In Case A the line joining the centers is at right angles to the vertical section plane. In Case B the line joining the centers lies in the section plane. The dotted lines show Case C. In this the unit mass is expanded into five spheres of the original size, each sphere possessing, therefore, one-fifth of a unit mass, and consequently a density of 0.020. In this case the five centers are arranged in a horizontal plane, the four outer spheres having their centers 50 km. from the center of the inner sphere.

The single sphere has a volume given by the formula $V=\frac{4}{3}\pi R^3$, in which $R=50$ km., and a density of 0.100. The three spheres have a volume therefore of $4\pi R^3$ and a density of 0.333. The three spheres make a solid of revolution whose semipolar axis is equal to $2R$, equatorial radius equal to R. Upon comparing this aggregate to a spheroid it is seen that the double density 0.667 of the intersecting portions compensates roughly for the two re-entrant zones on each side of the equator. To what regular spheroid does it approximate in its proportions? Let E be the equatorial radius of the spheroid and $2E$ the semipolar axis. The volume will be $\frac{8}{3}\pi E^3$, equal to the three spheres whose volume is $4\pi R^3$, or a single sphere of radius $1.44R$. Solving gives $E=1.14R$, $2E=2.28R$. The spheroid with these semiaxes is shown in broken lines in Fig. 9. This, then, is a spheroid which, if the density be taken as 0.033, is of exactly the same mass as the original unit sphere, or the three intersecting spheres, and which approximates in distribution of mass and in gravitative effect to these three spheres as shown in Fig. 9. The nature of the differences will be discussed later.

Case C shows five spheres of unit volume and of density 0.020 whose intersecting portions would consequently have densities of 0.040 and 0.060. In comparing the compound mass to an oblate spheroid these intersecting portions compensate roughly for the re-entrants between the spheres. The limiting dimensions of the

whole in the directions of the three principal axes are R and $2R$. Let it be required to find the value of the equatorial radius $2P$ and semipolar axis P of the oblate spheroid of equal volume in which the axes have these proportions. Then

$$\frac{16}{3}\pi P^3=\frac{20}{3}\pi R^3$$

$$P=1.08R$$

It is seen that the five spheres of density 0.020 have the same mass as a sphere whose radius is $1.71R$ and density 0.020; the same mass also as the unit sphere of density 0.100 and radius R and the oblate spheroid of density 0.020 and semipolar axis $1.08R$, equatorial radius $2.16R$. The vertical section of this spheriod is shown in dotted outline in Fig. 9. The distribution of mass and gravitative effect of the five spheres will be nearly the same as for such a spheroid. The nature of the differences, as in cases A and B, will be discussed later.

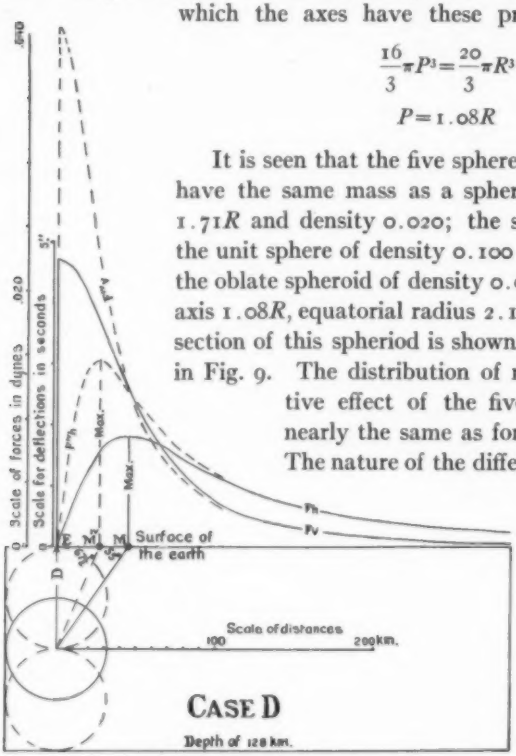

FIG. 10.—Horizontal and vertical components of the gravitative force due, first, to a sphere of 32 km. radius, density 0.100, depth to center 64 km., and second, to the same mass expanded into three such spheres with centers on a vertical line at depths of 32, 64, and 96 km.

The effect of the distribution of mass along a horizontal line and in a plane has been considered in cases A, B, C. There remains to be considered the effect of the distribution along a vertical line and in a vertical plane. Case D, Fig. 10, is given to show the effect of the distribution along a vertical line. The unbroken-line curves show the values of Fv and Fh for a sphere with density and depth corresponding to the unit sphere previously

used, but with radius of 32 km. instead of 50 km. The mass is consequently but 26 per cent of that of the unit sphere. Let this be expanded into three intersecting spheres with centers 32 km. apart and arranged in a vertical line. The composite mass will then extend from the surface to a depth of 128 km. and correspond closely in effect to a prolate spheroid with vertical axis.

The resulting values of the components of the gravitative force for these four cases which are of importance for the development of criteria may be tabulated as follows:

TABLE XXV

	Unit Mass		Ratio of Maximum Fv Divided by Maximum Fh	Value of θ for Maximum Fh
	Maximum Fv	Maximum Fh		
Sphere	0.085	0.033	2.6	55°
Case A	0.056	0.024	2.3	51±
Case B	0.056	0.023	2.4	38±
Case C	0.050	0.022	2.4	41±
	0.26 Unit Mass			
	Maximum Fv	Maximum Fh		
Sphere	0.023	0.009	2.6	55
Case D	0.041	0.015	2.8	67.5

To complete the series the curves should be drawn for five intersecting spheres arranged in a vertical plane analogous to Case V, first parallel and then at right angles to the plane of the section, making cases E and F, but the general character of the resulting curves may be inferred from the cases already given. Therefore, in order to abbreviate the discussion, an additional figure for cases E and F has been omitted.

It is seen from inspection of Figs. 9 and 10 and Table XXV that even for a constant mass and center of gravity the values of Fv and Fh change rapidly with the changing form of the mass. Upon the linear extension of a sphere into a form such as cases A and B the maximum value of Fv falls to two-thirds of its original value. For Case C it is still less. The ratio of the maximum value of Fv divided by the maximum value of Fh is also seen to change, but

more slowly, decreasing for the spreading-out of the mass in a horizontal direction, increasing for a linear vertical extension. For the spheroids whose axes are in the relation of 1 to 2 and to which the spheres are equivalent in volume, the changes would be still more marked. This is because the duplications of mass due to the intersecting spheres are near the center and in Case C a top view would show considerable deficiencies of mass in the equatorial zone between the spheres and the spheroid. These intersecting spheres are consequently more effective in producing gravity anomaly and somewhat more effective in producing deflection of the vertical in the zone of maximum deflection. For cylindrical disks of the same mass and proportions as the spheroids the changes away from the values for a sphere would be still greater, since a greater proportion of the mass would be removed from the center to the edges of the body.

Distinctive effects of individual spheres and spheroids.—In the subjection of deflection measurements to the hypothesis of isostasy Hayford found it necessary to consider the effects of topography to a distance of 4,125 km. and in the determination of gravity anomalies the topography of the whole earth and its compensation were considered. To what extent then do distant masses affect the local residuals and vitiate any attempt to analyze the effects of local masses? In answer, it is seen that the effects of distant masses are negligible. It is the great topographic contrast of ocean basins and continental platforms, to a lesser extent the large variations of relief within these areas, which require their effects to be considered to such a great distance. But it is found that this larger relief of the crust is nine-tenths compensated and the outstanding masses so far as known do not show any marked segregation as to sign within the continental areas as opposed to the oceanic areas. Furthermore, the unit areas departing markedly from isostatic equilibrium are much smaller than these major segments of the crust. Therefore where it is the effect of the outstanding masses which is under consideration they are seen to be individually small in comparison with the greater relief features of the globe, and further, they mutually cancel their effects. The local outstanding masses are furthermore of the same order of

magnitude as the more distant ones and therefore the effects of the distant masses sink to negligible quantities in comparison, in accordance with the law of inverse squares. The general agreement in the magnitude of the departures from isostasy, as shown by deflection residuals and gravity anomalies, shows, furthermore, that deep nucleal heterogeneities can have no large and broad regional effects, since such would affect the gravity measurements within a broad central circle to a greater degree than they would the deflections of the vertical. Therefore each region is seen to offer its local problems and the dominating centers of outstanding masses may be readily determined save where several such masses are contiguous, especially if of opposite sign. Let attention be given then to those features in the influence of outstanding masses which are indicative of the form and depth of the attracting mass.

Let the depth to the center of mass be D. Then it is seen that for ellipsoids near the surface in which the major axes are twice the minor axes the influence of form has mostly disappeared at horizontal distances from the epicenter or from $2D$ to $3D$, and at greater distances the curves become practically coincident. For greater departures from the spherical form the distances before the curves approach those given by a spherical mass are still greater. At these distances where the curves approach those of spheres, the effects of the form of the mass could not be distinguished, but the curves are so flat that neither could the effects of depth be readily evaluated. For instance, the curves of force at $4D$ to $6D$ for a sphere of mass M at depth D would be approximately the same as for a sphere of mass $4M$ at a depth of $2D$. Furthermore, at these distances from the epicenter the forces are so small in proportion to the maxima for the same mass that other outstanding masses would greatly change their value and prevent a correct analysis. Therefore, to be determinative, observations must be made at a number of points between the epicenter E and a distance not more than double that which at the point M gives the maximum value to Fh. It is seen that if a mass symmetrical about a vertical axis departs widely from the spherical form, this will be detected by noting the ratio of maximum Fv to maximum Fh, the latter being measured along any line radiating from the epicenter. If

the mass is unsymmetrical about a vertical axis, observations must be made along at least two lines at right angles to each other.

A minimum number of observations will define an isolated outstanding mass, but if several masses have their fields of force notably overlapping, a larger number of observations becomes necessary in order to differentiate their effects. An inspection of Figs. 9 and 10 shows that the shape of the curve of *Fv* between the epicenter and a distance where it falls to one-tenth the maximum value has more distinctive relation to the form of the immediately adjacent mass than has the curve for *Fh*. But the available geodetic data supply less information regarding the gradients of the gravity anomalies than for the deflection residuals. The latter are given along a certain belt of triangulation stations, whereas the gravity stations are located at long distances apart. Furthermore, but few of the gravity stations coincide with deflection stations. The present analysis will therefore rest upon the data giving the curve for *Fh*. This curve is flat at the top, so that the data will readily give the approximate value of the maximum but will not determine closely its distance from the epicenter. The value of θ will, however, be determined ordinarily on two sides of the epicenter by means of the deflection residuals and the mean will give a more reliable figure than either alone. But according to the form of the mass within those limits shown in Figs. 9 and 10 the value of θ may range from 38° to $67\frac{1}{2}^\circ$. If the abnormal mass is assumed to have a spherical form, its center will lie at an angle of 55° below the maximum value of *Fh* and at a depth 1.4 the distance to the epicenter. The error in locating the points of epicenter and maximum *Fh* may cause the estimate of depth to be in error 20 per cent and yet this figure will show definitely whether the sphere lies within the zone of compensation or in the centrosphere. If the mass, however, is in reality a horizontally elongate mass, the change in the distance *EM* from the epicenter to the point of maximum *Fh* in two directions at right angles to the epicenter will show that fact. A check on the form of the mass may be obtained if the value of the deflection curve is known with fair accuracy to a distance from the epicenter of three times the distance of the maximum. Let the distance to the point of maximum value

be M. The ratios of the value of Fh at M to the value at $3M$ are given below as measured from the curves.

RATIOS OF Fh AT M TO Fh AT $3M$

Sphere	2.4
Case A	2.1
Case B	4.3
Case C	3.4
Case D	2.0

It is seen that, from the difficulty in the precise location of M and hence the difficulty in locating a point as at $2M$ or $3M$, and furthermore the probability that other masses may influence to a degree not readily determinable the value at $3M$, this test cannot ordinarily be determinative. However, for spheroids with polar axes vertical, markedly prolate masses give a ratio distinctly smaller than for a sphere, the curve of deflections falling off more abruptly; markedly oblate masses, on the other hand, give a ratio distinctly larger than that for a sphere, the curve of deflection beyond the point of maximum being flatter.

If the outstanding mass approximates to an oblate spheroid with polar axis vertical, then the assumption of a spherical nature will locate the center of mass too deep and imply a greater mass than really exists. If, on the contrary, the form of the outstanding mass approaches the form of a vertical prolate spheroid, the interpretation of the deflections as caused by a sphere would locate the center of figure too high and give it too small a mass.

Suppose the curves for Fh shown in cases C and D have their maxima well determined in position and in magnitude, but that the values of the curves at distances two or three times beyond are not accurately known. Let these maxima be interpreted as produced by spheres. The depth and masses of the spheres which give these maximum deflections will be too great by the amounts shown in the following tabulation (Table XXVI, p. 462).

Where the data are sufficiently complete the form and depth of mass may both be determined, though a high precision is not to be expected. But in most cases with the present geodetic data the form of the mass will not be determinable and all that can be

done is to interpret the deflections as produced by spherical masses. What then are the geological suggestions as to whether vertical prolate or oblate forms may be expected to characterize the larger outstanding masses? In the one case the error of interpretation will be to make the masses appear too small and shallow; in the other case, to make them appear too great and deep.

TABLE XXVI

ERRORS DUE TO INTERPRETATION AS SPHERES OF UNIT MASSES AT DEPTH *D*

FORM	DATA		ASSUMPTION	RESULTING INTERPRETATION	
	Fk Max.	*EM.*	θ	Depth to Center	Mass
Case C (True)	4".5	74 km.	41°	1.0*D*	1.0*M*
Interpretation as a sphere	4.5	74 km.	55	1.6*D*	1.8*M*
Case D (True)	3.05	27 km.	67.5	1.0*D*	1.0*M*
Interpretation as a sphere	3.05	27 km.	55	0.6*D*	0.6*M*

Stocks, and especially volcanic pipes, approach in form to vertical cylinders, but these are merely connecting structures. On the other hand, mountain ranges and geosynclines, although linearly extended, are of breadth which is great in comparison with the depth of excess or defect of density. Laccoliths and regional extrusions are also broad in comparison with depth. The relations as regards the great intrusive masses are not so clear, but erosion exposes batholiths over progressively greater areas; and whole provinces which exhibit regional metamorphism give suggestions that they are underlain by widespread igneous bodies. The hydrostatics of the magmas and their differentiation into masses of unlike density would also give tendencies to layers and horizontal extensions of the larger masses of abnormal density. These would depart, then, from the form of spherical masses in the direction of oblate spheroids with their equators in a horizontal plane. Narrower belts of disturbance like that which passes through Washington, D.C., may, on the other hand, tend to have the form of vertical plates. Therefore in none but the smaller and connecting structures are there geological suggestions of vertical prolate form.

The summation of this discussion shows that if in the first assumption as to the form of abnormal masses they be taken as spheres, then the determination of the depth of the center of mass by means of the curve for Fh and the location of the point of maximum value is more likely to overestimate than underestimate the depths and masses. As the object of this investigation is especially to find the depth of masses and to test the hypothesis of centrospheric heterogeneity as a cause of deflection residuals and gravity anomalies, it is desirable to have the error of interpretation in the direction of indicating a depth too great rather than too small. Therefore the initial assumption that the outstanding masses are spheres is justified by the geologic probabilities and is found in Section B to be justified by the geodetic evidence. The next topic will therefore develop further the subject of the interpretation as spheres with the view to utilizing the geodetic data.

Depths of spheres whose epicenters are not on the line of traverse.— If the primary purpose of a geodetic investigation were the determination of the location of the epicenters of abnormal masses and then the measurement of their form, size, and depth, a series of gravity and deflection measurements could be made in a line passing above the mass and near the epicenter. The preceding discussion would then directly apply. In only a few localities, however, will a line of triangulation stations, located in connection with the measurement of the earth's surface, pass approximately over the center of a large outstanding mass. How then, from the locations and values of the deflection force along any linear belt of measurements, shall the location of the epicenter and depth to the center of an abnormal mass to one side of the line of traverse be determined?

In Fig. 11 is developed a method for the solution of this problem. In accordance with the preceding discussion and the reconnaissance nature of a first investigation, let it be assumed that isolated abnormal masses approach a spherical form; that is, that a mass may be regarded as concentrated at a point. Take the center of the mass as the center of co-ordinates and the axis X–X as lying parallel to the line of traverse. The epicenter is at E. Then the vertical distance from the center to the epicenter is D. The

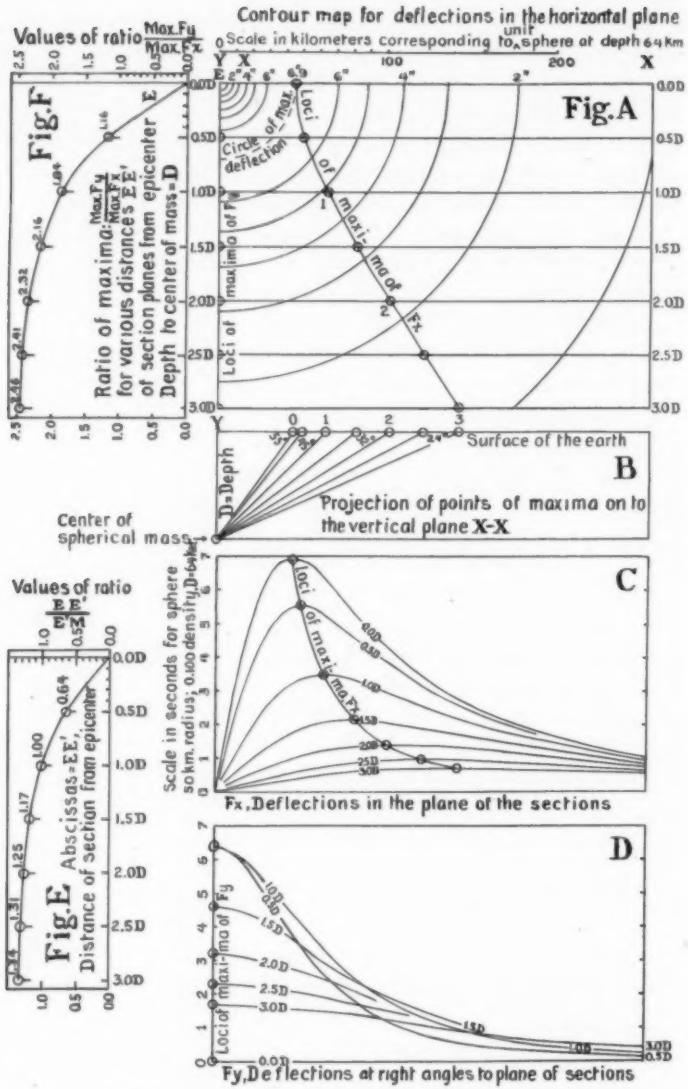
Contour map for deflections in the horizontal plane
Scale in kilometers corresponding to a unit sphere at depth 64 Km
Fig.A
Circle of deflection
Loci of maxima of Fh
Loci of maxima of Fx
Surface of the earth
B
D=Depth
Projection of points of maxima on to the vertical plane X-X
Center of spherical mass
Values of ratio Max Fy / Max Fx
Fig.F
Ratio of maxima Max Fy / Max Fx for various distances EE' of section planes from epicenter
Depth to center of mass = D
Values of ratio EE'/M
Fig.E
Abscissas = EE'
Distance of section from epicenter
C
Scale in seconds for sphere 50 km. radius; 0.100 density, D=0.50
Loci of maxima Fx
Fx, Deflections in the plane of the sections
D
Loci of maxima of Fy
Fy, Deflections at right angles to plane of sections

FIG. 11.—Values of the components of Fh on traverse lines which pass at various distances EE' from the epicenter E of a spherical mass at depth D; EE' being measured in terms of D.

distance EE' from epicenter to the traverse line is y. Then the co-ordinates of any point on the surface are x, y, and D. At any such point the component in the horizontal plane of the gravitative force due to the outstanding mass is Fh. But Fh is directed radially from all points to the epicenter. Fh, consequently, is subdivided into two components at right angles. Let that acting in the line of the section be called Fx, that at right angles be called Fy, parallel respectively to the X and Y axes.

The value of Fx for any point is given by the equation

$$Fx = dc\frac{4}{3}\pi R^3 \frac{x}{(x^2+y^2+D^2)^{\frac{3}{2}}}$$

and the value of Fy is given by the equation

$$Fy = dc\frac{4}{3}\pi R^3 \frac{y}{(x^2+y^2+D^2)^{\frac{3}{2}}}$$

Fig. 11 shows the results of the solution of these equations. In Fig. 11A is drawn a contour map of the deflection force produced by the unit sphere. The deflection force Fh at any point is measured by the contour map and is directed toward the epicenter. The lines of equal deflection are seen to be circles with center at E. They show in plan the values which were shown in section by the full lines for Fh in Figs. 9 and 10. The contours, as previously discussed, are seen to give the form of a volcano whose crater has a rounded rim and a conical interior reaching to the epicenter.

Now let a number of parallel sections be taken at horizontal distances from the epicenter equal to $0.0D$, $0.5D$, etc. The curves for Fx for each section are shown in Fig. 11C and for Fy in Fig. 11D. The points of maximum value for each section are indicated in A, B, C, and D and through these points are drawn the curves which are loci of maxima. For Fy there is a single maximum for each section and this is situated at $x=0$. For Fx there are in each section two equal maxima but of opposite sign, one for a plus, the other for an equal minus value of x. In Fig. 11C only one side of the curve is shown, that for plus values of x.

Fig. 11B shows the points giving maximum values of Fx projected onto the vertical plane passing through X–X. The dip

angle θ measures the slope from the point of maximum value to the center of mass. Here is shown its value when projected onto the plane X–X. This projected angle is seen to grow smaller with each more eccentric position of the section plane.

For section $0.0D$ the real value of θ is $55°$
For section $1.0D$ the projection of θ' is $45°$
For section $2.0D$ the projection of θ'' is $32°$
For section $3.0D$ the projection of θ''' is $24°$

Therefore it is seen that if the traverse line were assumed to pass through the epicenter of all disturbing masses, the error introduced would be to show the center of mass deeper than it really is. The nature, however, of the geodetic data permits this assumption to be eliminated and the distance EE' to the section plane to be approximately determined. An error up to $0.5D$ will not involve much error in the resultant depth as determined by the projection of θ. At each station along a line of triangulation[1] both Fx and Fy are determined and their resultant points toward the center of gravitative control. Each station gives an independent determination of this resultant and the intersection of two resultants if accurately determined and due to the gravitative force of a single symmetrical mass would give an accurate location of the epicenter, measuring its distance and direction from the traverse line. The data in many cases permit as many as three or four resultants to be drawn, the size of the triangle of their mutual intersections showing to what degree the forces may be ascribed to a single center. The relative positions of the line of section and epicenter of mass are thus in many cases approximately established.

But although the relative position of epicenters and traverse line are thus ascertained, the depth of the masses remains to be solved. In Fig. 11 the distance of the traverse line from the epicenter is given in terms of D, but this is the unknown. Two independent methods lead up to the solution of D.

First, on any line of section occurs a zero point for Fx. Let this be called E'. On each side of the zero point for Fx occurs a

[1] As shown in illustration No. 3, Hayford, *Supplementary Paper*.

maximum value for Fx. Let the point of this maximum be called M. These two values are given by the geodetic data. Then for any spherical mass the ratio of $\frac{EE'}{E'M}$ increases with increase in the ratio of $\frac{EE'}{D}$ but not as a rectilinear function. This relation of ratios is shown graphically in Fig. 11E, in which the abscissas are the values of $\frac{EE'}{D}$ and the ordinates are the corresponding ratios of $\frac{EE'}{E'M}$. This ratio may be determined from the geodetic data but *from the location of the maxima, not their amount.*

The second method for determining the depth depends upon the ratio of the maximum value of Fy for any traverse line to the maximum value of Fx for the same traverse line, thus being dependent upon the *relative values of the maxima and not their location.* This ratio also increases with increase in the ratio of $\frac{EE'}{D}$ but not as a rectilinear function. The maximum Fx for a section at any distance from E is shown in Fig. 11C. For example, if $E'E = 0.5D$ the maximum Fx for the unit sphere is $5.52''$. The maximum Fy is shown in Fig. 11D, and for $E'E = 0.5D$ is $6.42''$. The ratio of 5.52 to 6.42 is 1.16. These ratios are shown in Fig. 11F for all traverse lines up to a distance of $3.0D$ from the epicenter. The value of this ratio is given by the geodetic data for any traverse line and hence the distance to the epicenter is given in terms of D.

In conclusion on this topic it may be said that the curves shown in Figs. 11E and 11F are independent of the mass or volume of the sphere, depending only upon its depth, and are adapted to use with the geodetic data. It is seen from both curves that the significant ratios change in value rapidly with increasing distance of the traverse line from epicenter up to a distance $E'E = D$, but beyond this point the change in the value of the ratios becomes progressively small as compared to a change in the distance of the section plane. The method is therefore well adapted for

determining the depth of the outstanding masses assumed as spheres, provided the section line is not farther from the epicenter than the latter is above the center of mass. The method may be used, however, with less precision for distances of the traverse line $E'E$ up to $2D$. Beyond this distance, however, influences of other masses or errors in the geodetic data would be likely to give wholly erroneous results, not distinguishing between a large excess of mass at a great depth or a smaller one at a much less depth.

[*To be continued*]

CERTAIN TYPES OF STREAM VALLEYS AND THEIR MEANING

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A recent study of the "thalwegs" or immediate valleys of a number of streams, with particular reference to their ground-plan and their form as revealed in cross-section, has brought to my attention a wide difference in form of valleys produced by different streams and even by a single stream in different parts of its course. A subsequent failure to find in the literature any approximately complete explanation of these differences in form leads me to attempt the following analysis of the work of a river in carving its valley, in the hope that it will lead to a clearer understanding of the meaning of these marked morphological differences and will pave the way for the use of these distinctive valley forms as criteria for the interpretation of the physical history of a region where they occur.

An examination of a large number of valleys, as delineated on the contour maps published by the United States Geological Survey and by the French and German Surveys, led to their classification under three types which seem fundamental. For each type there is a further series of stages marking the development of the valley from its initial to its final form. For the three types I suggest the following names:

1. The Open Valley.
2. The Intrenched Meander Valley.
3. The In-grown Meander Valley.

The *Open Valley* is one which is either straight, as valleys go, or winding in broad, open curves. The valley sides are relatively straight and may be smoothly trimmed. The stream swings from side to side in broad, open curves which, except in the very earliest stages, do not necessarily correspond with the curves of the valley as a whole.

The *Intrenched Meander Valley*[1] is one whose stream, having inherited a meandering course from a previous erosion cycle, has sunk itself into the rock with little modification of its original course.

The *In-grown Meander Valley* is one whose stream, which may or may not have inherited a meandering course from a previous cycle, has developed such a course or expanded its inherited one as it cut down. Thus, as the stream sunk its channel lower and lower into the bed-rock, the meanders were continually *growing* or expanding. The term "in-grown" has been chosen to express this idea.

Three valleys illustrating the above types have been selected from the United States Topographic Atlas. A brief description of each should bring out the most essential characteristics of its type:

1. Typical of the first group, the *open valley*, is that of the Kanawha River as depicted on the Charleston (W.Va.) special sheet; U.S. Geol. Survey (see Fig. 1). The valley of Elk River, entering the Kanawha at Charleston, also illustrates this type as developed by a smaller stream.

The "thalweg," or immediate trough of the Kanawha River, is sunk some six or seven hundred feet below the general level of the neighboring upland, which it traverses in broad, open curves. The valley bottom is flat and about two-thirds of a mile wide, or about six times as wide as the stream. The sides are steep and trimmed to remarkable regularity—being, in fact, practically parallel. There is a slight tendency to greater steepness of the concave bank on the sharper bends. In this open, flat-bottomed valley the stream swings in broad, free curves, now hugging one bank, now the other. A space of something like six miles intervenes between successive points of impingement against the same bank. The curves of the stream in its swinging do not everywhere correspond with those of the valley trough.

The flat bottom, so conspicuous in the valley of the Kanawha, is not considered an essential feature of the open valley. A non-meandering valley with a V-shaped cross-section would, in the sense

[1] The term "incised meander" has, apparently, been used synonymously with intrenched meander in previous writings. Would it not be well to use "incised meander" as a generic term covering both the above-described cases, and to restrict the meaning of intrenched meander as indicated above?

in which the writer proposes to apply the term, be as truly an "open valley" as the flat-bottomed one described above.

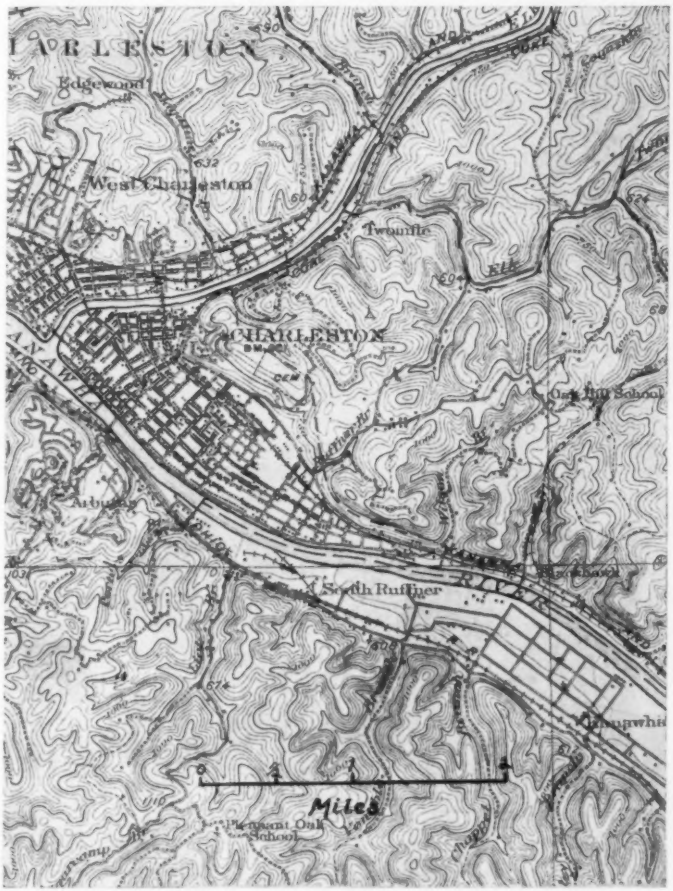

FIG. 1.—Part of the Charleston (W.Va.) special topographic sheet showing typical "open" valleys. Contour interval 20 feet.

2. The second type, or *intrenched meander valley*, is finely illustrated by that of the Kentucky River and its tributary the Dix,

as shown on the Harrodsburg (Ky.) topographic sheet. A part of this is reproduced as Fig. 2.

FIG. 2.—Part of Harrodsburg (Ky.) topographic sheet showing typical intrenched meander valleys. Contour interval 50 feet.

These rivers follow remarkably sinuous meandering courses. The meanders, however, are sunk sharply into the upland, and

show no evidence of having been widened appreciably during the time of their incision. The windings of the valley correspond with

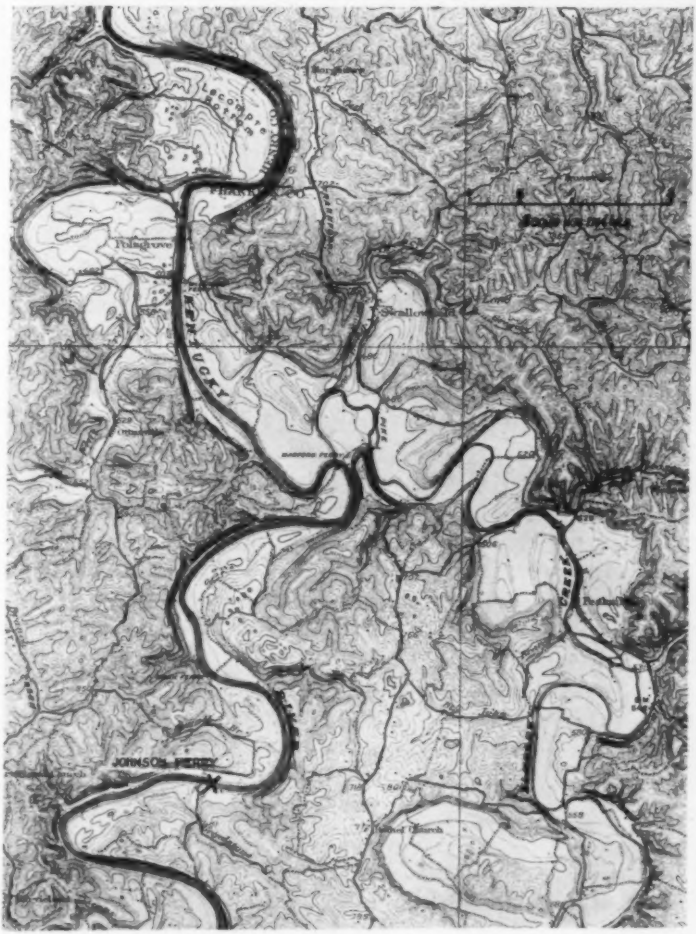

FIG. 3.—Part of Lockport (Ky.) topographic sheet showing typical in-grown meander valleys. Contour interval 20 feet.

those of the stream. Lack of flats along the stream indicates that incision is still in progress.

It is especially characteristic of this type that the portion of the upland between the meander loops keeps its full height and flat surface almost to the inside end of the loops. The valley, in cross-section through the end of one of the loops in a direction transverse to the general direction of the stream, is practically symmetrical. There is a slight tendency to the development of "slip-off slopes" and undercut bluffs, but it is subordinate.

3. The *in-grown meander valley* is well shown on many of the United States topographic maps. One of the most beautiful which has come to my notice is portrayed on the southern part of the Lockport (Ky.) sheet, where the valleys of the Kentucky River and Elkhorn Creek display with great clearness the characteristics of the type (Fig. 3). The concave and the down-valley banks are marked by steep bluffs, while on the insides of the loops are well-marked slip-off slopes descending gently to the river. This characteristic feature is clearly shown on the inside of the loop just above Johnson Ferry (**X** on the map, Fig. 3).

Such a valley presents clear evidence that whatever may have been the course of the river in a previous cycle, the meanders have been much widened and enlarged during the present cycle.

All three of the valleys described above are the products of comparatively young and decidedly vigorous streams. All three, moreover, are in approximately the same stage of development in the erosion cycle—namely, youth to maturity. All are, or have recently been, degrading streams. We cannot, therefore, ascribe the differences in valley form to differences in the stage of development of the valleys. The differences are more fundamental than that. Nor are we dealing with old-age streams in which the meanders are features of the flood plain. Such old-age streams and valleys may be looked upon as *end products* of the development of any one of the types of valley described above.

Having recognized these distinctive valley types we are confronted with the problem: What conditions determine the type of valley which a stream will develop? A satisfactory solution of that problem calls for a brief review of some of the basic principles of river work.

ANALYSIS OF THE WORK OF A RIVER IN CARVING ITS VALLEY

The development of the immediate valley or thalweg of a river is accomplished mainly by a combination of three well-known processes—vertical down-cutting, lateral cutting, and down-valley migration of the curves or meanders, to which Davis has applied the excellent term "sweep." Of these, vertical cutting, when the conditions are favorable, is much the most rapid, because it has gravity as its direct and powerful ally.

In a normal degrading stream, all three processes are active in shaping the valley, but vertical cutting may be so much more rapid than the others as to mask their effects. When, however, a stream approaches grade and vertical cutting diminishes, one or the other of the two remaining processes advances to prime importance.

Any one of these processes, if dominant, is capable of impressing upon the valley, which it is helping to shape, certain characteristics of form which are distinctive. An acquaintance with these distinctive forms should enable one, on examining a valley, to determine which process took the leading part in its sculpture. If, carrying the analysis back a step farther, we can determine the controlling factors which lead to dominance of a given process, we shall be in a position to read much of the physical history of a region from the form of its valleys.

DISTINCTIVE FORMS ASSOCIATED WITH EACH OF THE THREE VALLEY-CUTTING PROCESSES

Dominant down-cutting implies that a stream is sinking its bed vertically at a greater rate than it is cutting laterally or down-valleyward on the bends. The necessary result of such a process is the development of a *gorge-valley* with narrow bottom, not, as a rule, much wider than the stream at flood, and with sides whose steepness, while usually great, depends largely on the nature of the rock and on the activity of weathering agents. The stream tends to sink itself vertically in whatever course, whether straight or meandering, it may hold at the beginning of down-cutting. Lateral cutting and sweep will modify this original pattern somewhat, but not to the extent of destroying the characteristic gorge form.

Dominant lateral cutting.—The well-known tendency of a stream to swing from side to side of its valley results in very unequal wear on its channel—active under-cutting on the outsides of the bends being accompanied by deposition on the insides.

An inspection of Fig. 5 will make clear that lateral cutting alone, a result of the force due to inertia, should result in *symmetrical* enlargement of the meanders without their displacement. The resulting topographic feature would be the symmetrical undercut bluff or meander scaur combined, if the stream were cutting down meanwhile, with symmetrical slip-off slopes. A thalweg, therefore, shaped by dominant lateral cutting should present a pronounced scalloped outline—the greater the dominance of lateral cutting the more symmetrical and the more nearly circular the scallops. That we do not find these symmetrical forms in normal valleys is sufficient evidence that with lateral cutting there is combined another process which destroys the symmetry. That process is *sweep*.

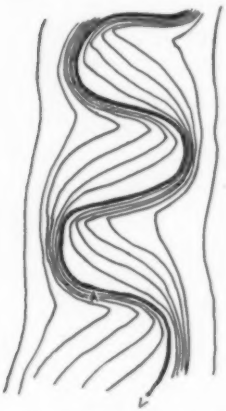

FIG. 4.—Contour sketch showing typical features of an in-grown meander valley, and the effect of sweep in producing assymmetry. Note the unsymmetrical necks of land on the insides of the loops; the sharp undercut bluffs on the outside and down-valley sides of the bends, and the gently inclined slip-off slopes opposite.

Dominant sweep.—The down-valley migration, or sweep, of meanders brings the stream successively over all parts of its valley flat. Any obstructions in the valley, opposing the orderly march of the meanders across and down the valley, is attacked. Thus the whole tendency of sweep is toward a *clearing-out* of the valley by the removal of spurs or other obstructions. It follows, therefore that, where sweep dominates, only the open type of valley (type 1 above) is "stable." In the case of a valley of the meandering type, the tendency of the meanders to sweep down-valley leads to more active erosion on the down-valley sides of the meander bends than on the up-valley sides. This results in asymmetry of both

undercut bluffs and slip-off slopes with resulting forms like those illustrated in contours in Fig. 4.[1]

Combinations of processes.—As already stated, all three of the above processes are normally in operation at the same time along a stream valley. The analysis of the form produced when any one is dominant is a necessary preliminary to an understanding of the more complex forms resulting when two or more of these processes act in combination. Where down-cutting is vigorous the effects of the other two are, as a rule, masked to a greater or less degree. A gorge-like valley is the result. When, in the history of a stream, down-cutting ceases or becomes very slow, lateral cutting continues to widen the valley[2] while sweep tends to clear it out if it is not already relatively straight. Once a valley assumes the open form, whether as a result of initial conditions or as a consequence of the clearing effect of sweep, the constant down-valley procession of meanders tends, throughout the remainder of the cycle, to maintain this form by shifting continually the locus of attack of the stream, and to produce a valley with fairly uniform width and with relatively straight walls.

Between the stage marked by predominant down-cutting and that marked by predominant sweep there must be an intermediate stage when sweep, down-cutting, and lateral cutting bear such a relation to one another that whatever width of meander is gained by the stream in its lateral cutting is retained because of the contemporaneous down-cutting, while the spurs between the meanders, with their characteristic slip-off slopes, are not cleared out by sweep. In valleys developed under such conditions, the slip-off slopes would be conspicuous and the undercut bluffs well developed. The form would be that of the in-grown meander valley (type 3 above). Sweep, in this case, would be subordinate because, as will be explained more fully in a subsequent paragraph, in the interval between the sweep of successive meanders past a given point,

[1] W. M. Davis, "Incised Meandering Valleys," *Bull. Geog. Soc. Philadelphia*, IV (1906), 182–92.

[2] A. Penck, *Morphologie der Erdoberfläche*, II. Buch, I. Abschnitt, S. 350: "Langsame Vertiefung ist das Erforderniss zum gleichseitigen Verschieben der Flussbetten. Sobald erstere rasch geschieht und auch letztere erfolgt, so wird eine ungeheuere Trummermasse dem Flusse zugeführt werden, die er nicht zu bewältigen vermag."

the stream would have lowered its bed so that each succeeding meander in its sweep would be opposed by rock.

CONTROLLING FACTORS DETERMINING THE DOMINANT PROCESS

Down-cutting will dominate whenever a stream finds its bed well above its local profile of equilibrium. After a stream reaches grade it may still lower its bed as the general level of the country is lowered, but the process must be gradual—too gradual for the continued dominance of down-cutting over lateral cutting or sweep.

Lateral cutting.—The tendency toward lateral cutting, which, it appears, is dependent upon a balance between two opposing forces[1]—(1) that due to inertia, tending to displace the thread of fastest current tangentially from one bank to the other, thereby favoring lateral cutting, and (2) the down-stream component of gravity, tending to make the current follow parallel to the walls of the channel—is controlled largely by the gradient of the stream, which determines the latter of these two forces. The lower the gradient the greater the proportion of the stream's energy used in lateral as distinguished from down-valleyward corrasion.

Volume also must play a part, for increased volume, by decreasing friction and thereby increasing velocity, would augment the value of the force due to inertia, and would therefore favor lateral cutting.

Sweep, it appears, particularly the ratio of the rate of sweep to that of down-cutting, plays a most fundamental part in determining the form of the valley. It deserves, therefore, a more detailed study.

Three factors suggest themselves as determining the rate of down-valley migration of meanders, or sweep. These are: the gradient of the stream, its volume, and the character of the material against which it impinges at the bends.

The importance of gradient and volume will be apparent from a study of Fig. 5, which illustrates the forces acting in a meandering stream. In a bend such as that shown in the diagram, inertia, if acting alone, would throw the current from *A* diagonally across the channel to *B* in the direction of the tangent to the curve

[1] See Fig. 5.

of the bank at *A*. If only inertia were acting, the meanders would be enlarged symmetrically; but there are two other factors to be taken into account. These are the down-stream component of gravity and the tendency of the thread of the current to follow the shortest course round the insides of the bends.[1] The former of these acting on a particle of water at any point, *X*, would tend to make the particle move in a direction *X-Y*, parallel to the median line of the stream at that point. The latter would tend to lead the current along the inner bank, *A-C*. The resultant course of the thread of fastest current would be along some line such as *A-D*, intermediate between *A-B* and *A-C*, giving greatest erosion at and below *D* rather than at *B*. The current, on account of a combination of the two latter forces, would hug the down-valley bank *A-C* more closely than would otherwise be the case, while it would draw away from the up-valley bank near *B*. Thus would the meander migrate down-valley.

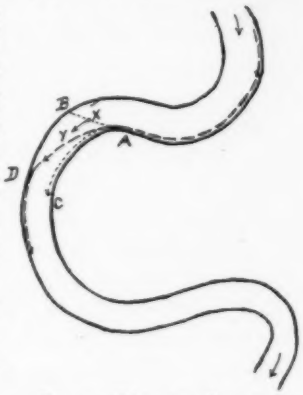

FIG. 5.—Diagram of a stream meander to show the explanation of the down-valley migration or sweep of meanders and the effects of gradient on the forces concerned. The dashed line represents the thread of fastest current; the dotted lines the courses this thread of fastest current would take if the various forces were acting alone. The line from *A* to *B* represents its course if inertia alone were active; *A* to *C* that if inertia were inoperative.

It is at once evident that an increase in gradient of the stream would increase this tendency to down-valley migration because it would increase the down-valley component of gravity at a greater rate than it would increase the force due to inertia, since the latter, dependent as it is on velocity, would be cut down by friction.

Increase in volume without change in gradient would tend toward greater symmetry and relatively slower down-valley migration, for greater velocity, due to lessened frictional resistance,

[1] J. Thompson, *Proceedings of the Royal Society of London*, XXV (1876), 5–8.

would increase the inertia component without altering either of the other two.

In respect to the third of the factors determining the rate of sweep, it is of course evident that this rate will, other things being equal, depend directly on the resistance of the material against which the stream is working. If this material is the soft alluvium of a flood plain, the meanders will sweep past rapidly; if it is resistant rock they will go much more slowly. If the banks are low, less material must be moved than if they are high, consequently the meanders will tend to move more rapidly.

Where the meanders are intrenched, lateral, vertical, and down-valley erosion will be equally affected by the character of the rock, but where a stream is flowing on a flood-plain-floored valley with rock bluffs on the sides, lateral cutting into the valley walls is opposed by rock, while sweep is opposed only by soft flood plain alluvium of moderate thickness. The result must be a relatively rapid down-valley migration of the curves of the river.

Conditions under which sweep becomes dominant.—The ratio between the rate of sweep and that of down-cutting is one of the fundamental factors in determining the form of a valley. Three common conditions lead to dominance of sweep. These are: (1) a stream at grade in a relatively straight valley; (2) a stream which for any reason, for instance a decrease in gradient, has ceased down-cutting altogether; and (3) a stream carrying *coarse material*.

In the first of these cases a stream, having cut down to its profile of equilibrium in a relatively straight valley, still continues to deepen this valley, but only slowly, as the region as a whole becomes lower. The stream swings from side to side in ever-broadening, open curves. The tendency to down-valley migration of these curves is opposed only by the resistance of the thin coating of relatively weak flood-plain material on the valley flat, plus that of the thickness of rock which represents the amount of down-cutting in the interval between the passage of successive meanders. The tendency toward lateral cutting is opposed by the *rock* of the valley walls. The result is that when the stream is turned against the valley side, the flood-plain material on the lower side of the bends

yields rapidly and the locus of attack on the rock of the valley side is shifted down stream before any great impression on the valley walls can be made. A repetition of this process with its constant down-stream migration of the locus of attack must produce relatively straight valley walls.

In a stream of constant volume, as the gradient gradually decreases with advancing age, the tendency to sweep (dependent on the gradient) decreases, while the tendency toward lateral expansion of the meander loops increases (since the latter becomes relatively more effective as the gradient becomes less). The stream, therefore, remains against its rock banks at any given place longer than before with the result that the straight valley wall characteristic of its earlier stages gradually gives way to a series of crescentic scaurs marking the successive points of impingement—the older ones, perhaps, considerably modified by general weathering since their formation.[1]

In the second case, when a stream has ceased down-cutting altogether or has so nearly ceased that the amount of such cutting is at a minimum, down-valley sweep, combined, of course, with lateral cutting, will take first place and will work out its characteristic results whatever may have been the original form of the valley. In valleys of either the intrenched or the in-grown meander type the stream will develop a flat flood plain, first on the up-valley sides of the bends and on the convex banks of the loops; gradually it will cut off any spoon-shaped necks of land between the loops, perhaps producing "rock islands" as a by-product, and finally it will remove all spurs or isolated rock masses.[2]

The third case, streams carrying coarse material, is best illustrated by the small headwater tributaries of many streams, particularly those in regions of resistant rocks. A plentiful load of coarse material means a high gradient, for only a swift stream is "competent" to the coarse débris.[3] The high gradient gives high values to the forces responsible for down-valley sweep while the

[1] This feature is well shown on the Williamstown (N.C.) topographic sheet.

[2] See W. M. Davis, "River Terraces in New England," *Bull. Mus. Comp. Zoöl.*, XXXVIII (1902), 281–346.

[3] Gilbert, *Henry Mountains*, chapter on "Land Sculpture."

heavy load of coarse débris prevents rapid down-cutting. Sweep therefore dominates.

Conditions unfavorable for rapid sweep.—The following conditions are unfavorable for rapid sweep: low gradient, as explained above; relatively rapid down-cutting; and an original meandering course sunk into bed-rock (the latter not, of course, affecting the tendency to sweep, but only its absolute rapidity in a given case).

Under the second of these, relatively rapid down-cutting, the stream is continually cutting deeper into bed-rock and thus constantly increasing the amount of material to be moved by a meander in sweeping down-valley; not only so, but in the interval between the passage of successive meanders, the stream, at a given point has deepened its bed and there is still *rock*, not flood-plain material, to be removed by each succeeding meander as it sweeps down the valley.

In cases where down-cutting is long continued, but slow enough so that lateral cutting is conspicuous, there may result a peculiar type of valley with its bottom section exhibiting fine specimens of in-grown meanders, but with the upper part of its walls relatively straight. Such a condition would be the necessary result of a passage down-valley of a series of meanders during a slow uplift or its equivalent. What may be a valley of this sort is illustrated by the Chaquaqua Canyon as depicted on the Mesa de Maya (Colo.) sheet (Fig. 9).

The influence of load.—A full load of sediment, by occupying all the stream's available energy in its transportation, is an effective check to down-cutting. This does not, however, interfere with the process of sweep on the flood plain, for the sediment derived from cutting on a concave bank may be redeposited on the convex bank next below without in any way increasing the permanent load of the stream. It is a question also if it entirely prevents lateral cutting on the valley walls at the point of impingement of the meander bends. In a fully loaded stream, at any rate, sweep is dominant and the straight-walled valleys are to be expected.

DEDUCTIVE STUDY OF THE EFFECTS OF THE RATE OF UPLIFT OF A DRAINAGE BASIN ON THE FORM OF ITS VALLEYS

We have already seen what a fundamental rôle in the determination of valley form is played by the *rate of down-cutting*. A deductive study of the effects of differing rates of uplift, with their correlated differing rates of down-cutting, on streams of varying characters should enable us to evaluate properly this important factor.

Rapid uplift of a relatively straight stream.—Assume a region of perfectly homogeneous rocks, drained directly to the sea by a master stream and its tributaries. Let the region have advanced to the old-age stage in the erosion cycle, with all the streams thoroughly graded and with the master stream swinging in long, open curves on its way to the sea. Now, considering only the master stream, assume an uplift of the land uniform in amount throughout the drainage area, and so rapid that the stream in its down-cutting cannot keep pace with it. What will be the result?

In the first place, a wave of down-cutting, beginning at the mouth of the stream, will progress up the valley. Since, by assumption, the stream is a large one, down-cutting will be rapid and will greatly outstrip lateral cutting and sweep. The stream will quickly intrench itself in whatever course it happens to be following. Until grade is reached on the cessation of the uplift, deepening of the lower course of the stream will greatly exceed lateral cutting. As soon, however, as grade is attained, lateral swinging and sweep will increase in relative importance. The latter, as explained above, will dominate and the constant downstream procession of more or less open meander loops will widen the valley and at the same time tend to produce relatively straight valley walls. Further widening and further decrease in gradient will, as already explained, result in the scalloping of the bluffs. Finally, as the flood plain comes to exceed the width of the meander belt, the further widening will be more irregular, and the older of the meander scaurs will lose their original sharpness, though not their characteristic ground-plan.

The valley resulting from such a series of events would correspond to our "open valley" (type 1) and the series of stages through

which it would pass in its development ought to be matched in actual valleys subjected to similar conditions.

Rapid uplift of a meandering stream.—Assume all initial conditions the same as before except that the master stream instead of flowing relatively straight, writhes in closely looped meanders along its lower course toward the sea. Assume, as before, a rapid uplift terminating after a moderate interval.

Down-cutting, as before, will greatly exceed lateral cutting and sweep and the stream will intrench itself with little modification into the bed-rock. On the sharper bends the accentuated lateral cutting may succeed in producing under-cut bluffs and slip-off slopes of moderate extent, but as a whole the stream will merely intrench itself in its inherited course. The cross-section of the valley, except at the sharpest bends, will approximate symmetry, and the upland marking the original plain will extend unbroken into the inside of the meander loops, in sharp contrast to the gently decending slip-off slopes which mark the insides of the bends when the meander has *developed* as uplift proceeds.

As uplift and down-cutting continue, some of the narrowest necks between meanders are likely to be cut off by under-cutting as has happened at the Frying Pan Bend, and as may eventually happen just above Handy's Bend in the Kentucky River (Fig. 2).

Just so long, however, as the gradient of the stream remains so great that down-cutting dominates, will the meanders continue to intrench themselves deeper and deeper, still holding nearly their original courses.

When the uplift ceases, the master stream will quickly cut down to grade. Thereafter deepening will be slow. Lateral cutting combined with sweep will at once advance to the dominant place, and the bottom of the valley will become widened. The outside and down-valley sides of the meander bends will feel the effects of the changed régime first and most strongly; flats will develop; narrow necks like that at Handy's Bend (Fig. 2) will be quickly cut through, making "rock islands" of the cut-off remnants. Down-valley sweep will continue active and will tend to remove, gradually, the spurs and rock islands from the valley and to transform it into a flat-bottomed, crescent-bordered trough. In

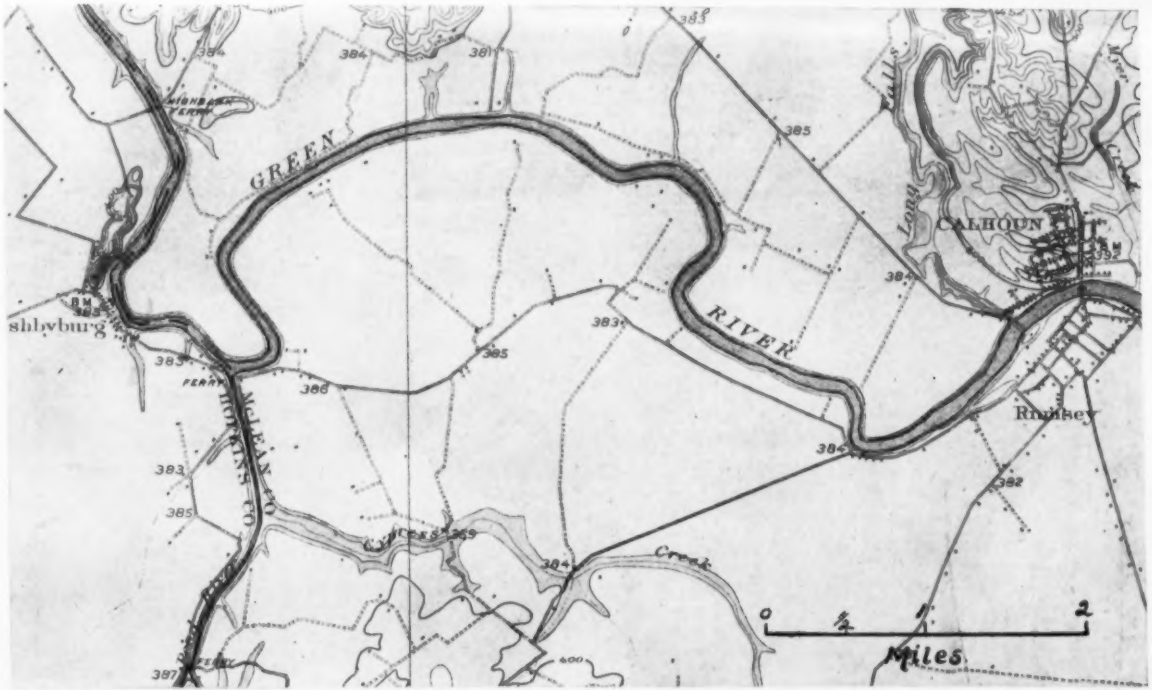

FIG. 6.—The earliest stages of the formation of an intrenched meander valley. Note the almost complete absence of evidence of lateral shifting of the meanders. From the Calhoun (Ky.) topographic sheet. Contour interval 20 feet.

the valleys of swift streams the latter process will be relatively rapid; in those of streams of low gradient, such as the master stream we have postulated, the river may flow for long ages before removing all signs of its former stages.

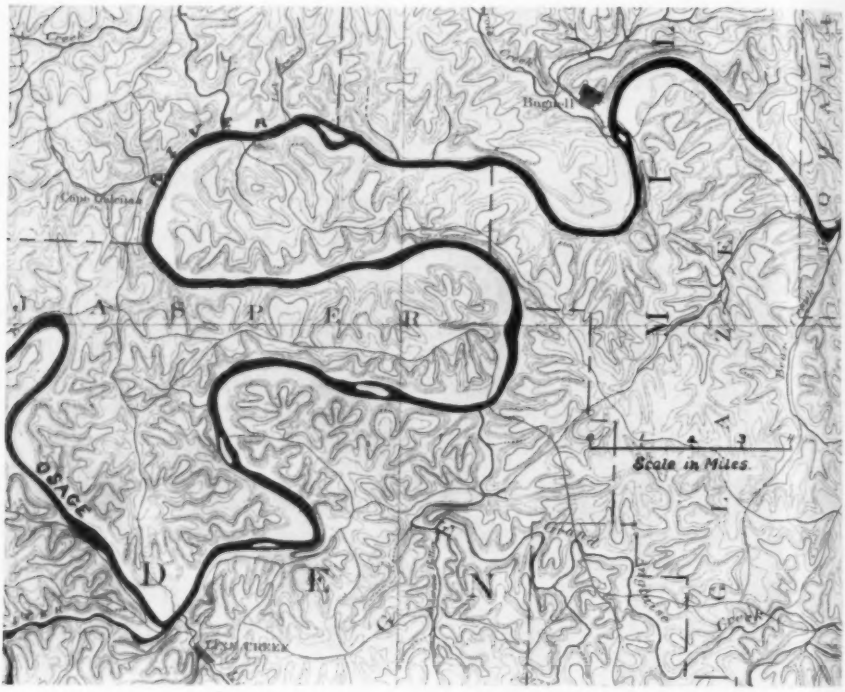

FIG. 7.—An intrenched meander valley in which the stream, having reached grade is beginning to widen the valley. Note that the upland between the loops keeps practically its full height almost to the end. From the Versailles (Mo.) topographic sheet. Contour interval 50 feet.

Examples of streams which show features corresponding to those just deduced are very numerous. For the earliest stage, when intrenchment is just beginning, the Green River in Kentucky (Fig. 6, Calhoun sheet, U.S. Geol. Surv.) is instructive. Here the predominance of down-cutting over lateral cutting in the determina-

tion of the valley form is very evident. Later stages, where the trenching is deeper but where the down-cutting is still active, are shown by such streams as the Kentucky and the Dix (Fig. 2). A still later stage, when grade has been reached and the formation of a flat valley-bottom has begun, is excellently shown on the Versailles (Mo.) sheet of the Geological Survey (Fig. 7). A still more advanced stage in a stream which seems to have had a history somewhat like that just outlined is shown by the Black River on the Oberlin (Ohio) sheet (Fig. 8). At the beginning, the stream, in its lower reaches, seems to have had a distinctly meandering course in which it cut down quickly to grade. It is at present changing the form of its meander bends and opening out its valley on account of a relatively rapid down-valley sweep. At the town of Elyria, where the stream has evidently encountered rock of considerable resistance, it is still cutting down and holds its original meandering course only slightly modified. The valley here is a typical intrenched meander in its youthful stage of development. Further down, in the softer rocks, a more advanced stage has been reached.

Slow uplift of a straight stream.—The results of another type of uplift may be deduced by assuming, as before, an ideal drainage system with the stream flowing in broad, open curves, to be subjected to an uplift so slow and continuous that the stream, while forced to continued down-cutting, is, nevertheless, able to maintain itself continuously near grade.

Under such conditions lateral cutting will assume an important rôle from the very first, and the stream, though its original course may have been comparatively straight, will come to swing in ever-broadening curves. Continued slow lowering of the stream bed will insure the retention of all width of meander gained by lateral cutting. Sweep will, for reasons already set forth—namely low gradient and continuous cutting in rock—take a subordinate place and will not hinder the further development of the meanders in the bed-rock. As the process continues the stream will sink its bed deeper and deeper into the rock; the meanders will become wider and wider, and the form of the resulting valley will be characteristic—the outsides of the meander bends marked by steep undercut bluffs and the insides by more or less gently inclined

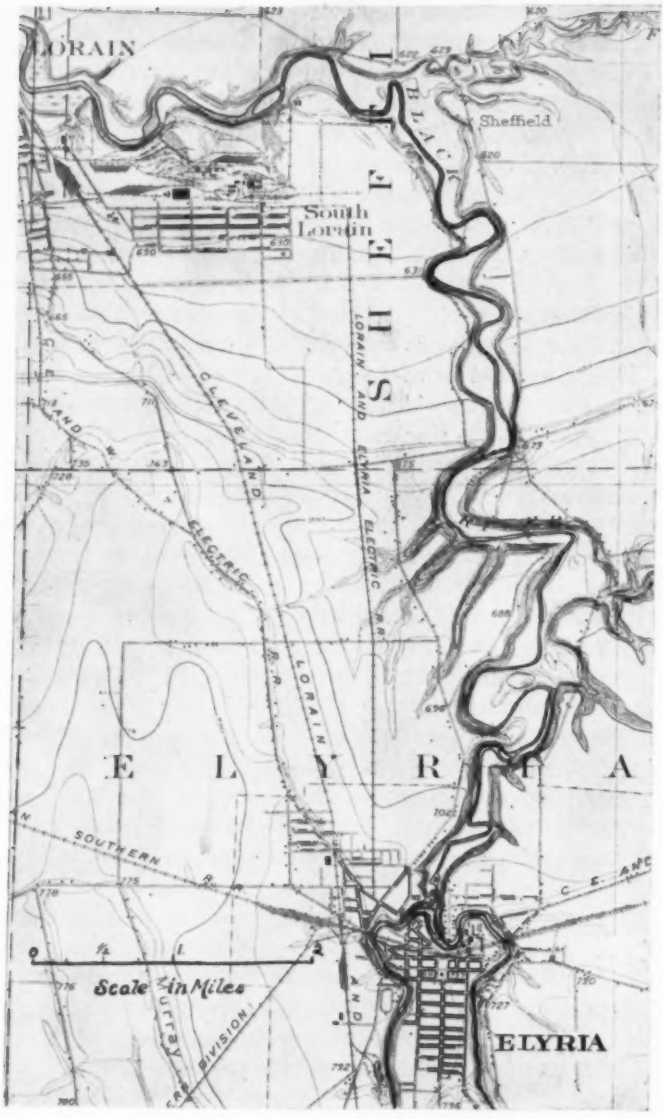

FIG. 8.—Part of the Oberlin (Ohio) topographic sheet, showing a meandering stream of the intrenched type in two different stages of development in different parts of its course. At Elyria it is in the earliest stage of intrenchment; farther down, grade has been reached and sweep is clearing out the valley.

slip-off slopes. Sweep will lead to asymmetry of the necks of land between the loops.[1] In time, if the uplift is continued, a single meander may sweep more than its width down stream, but before the next one arrives the stream bed will have been so lowered that the characteristic valley form will still be preserved. In time cut-offs may develop in solid rock. As uplift continues and the valley becomes deeper, the amount of material to be removed by lateral cutting, or as a result of lateral cutting, will become so great that further increase in the width of the meander belt may be checked. The valley form may then approximate that shown by the Chaquaqua Canyon, Mesa de Maya sheet, Colo. (Fig. 9).

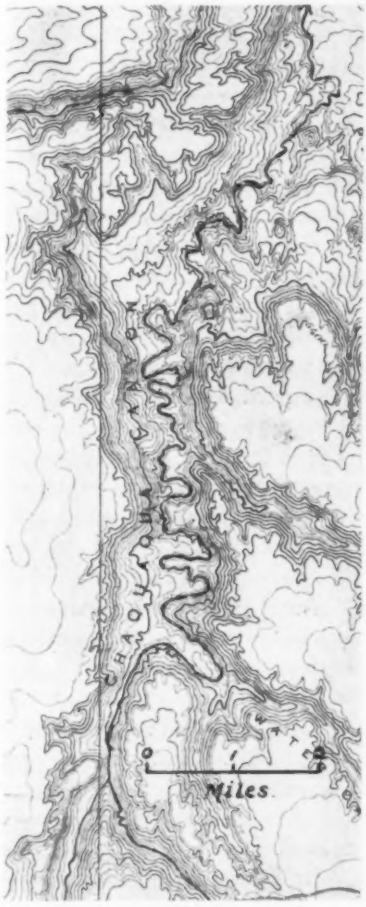

FIG. 9.—A deep valley of the in-grown meander type at the bottom; upper walls noncommittal. From the Mesa de Maya sheet, Colorado.

The reader has, doubtless, ere this discovered the correspondence between these deduced consequences and the features of the ingrown meander type of valley as described above (see also Fig. 3).

After such a slow uplift, with its consequences as just described, has progressed until the stream is incised to a considerable depth below its

[1] W. M. Davis, "Incised Meandering Valleys," *Bull. Geog. Soc. Philadelphia*, IV (1906), 182–92.

original level, what will be the consequences of a cessation of uplift?

The stream, which as we have assumed has kept an approximately graded condition throughout the uplift, will soon cease active down-cutting. Lateral cutting and sweep will, however, continue. A flat flood-plain will begin to develop. All meanders will be pushed down stream, some probably being cut off in the process, leaving rock islands standing isolated on the flats. The tell-tale slip-off slopes and undercut bluffs might, nevertheless, remain in protected spots, and, at least until the new régime became far advanced, would tell the story of the mode of development of the valley, and, incidentally, of the slow and continuous character of the down-cutting. Should the land remain long enough at one level all traces of the undercut bluffs and slip-off slopes would finally be obliterated, and the stream would swing in its meanders across a flat, open flood plain bordered by meander-scalloped valley walls, or bluffs.

Thus we see that the *end product* in this case is similar to that in the others and that the distinctive valley types are characteristic only of the youthful and mature stages of development, while all tend to become alike in the old age stage.

Slow uplift of a meandering stream.—If, with initial conditions as in the last case, we assume, instead of a straight stream, a meandering one with uplift slow enough to keep the master stream cutting down, but never to its full capacity, we should expect lateral cutting, as before, to play a relatively important part from the first. The original meanders would increase in width as they intrenched themselves into the rock. Cut-offs would probably occur. The form of the valley would be that of the *in-grown meander valley* with undercut bluffs on the outsides of the meander bends, or concave banks, and slip-off slopes on the insides or convex banks. There would probably, however, be some of the features of the intrenched meander such as the flat upland between the original meander bends. In the earlier stages of the development of such a valley it might be possible to determine definitely whether the original stream was meandering before the uplift by noting whether or not slip-off slopes

occupy the greater part of the width of the necks of land between the loops.[1]

Conclusions from deductive study.—From the foregoing considerations it becomes apparent that a *slow* uplift, of such a master stream, unless it be so slow that sweep dominates, will tend toward the formation of the *in-grown* type of meander, whether or not the stream meandered widely in the cycle preceding the uplift, and that a *rapid* uplift will result in the intrenchment of the stream in whatever course it may happen to be holding at the time. If this original course happens to be relatively straight, a valley of the open type will develop; if it is meandering, the meanders will be intrenched but will present characteristics sufficient to distinguish them from the *in-grown* meanders resulting from *slow* uplift.

It appears also—and this fact should be emphasized—that, for each of the hypothetical cases studied, the form of the valley goes through a definite and characteristic evolution as it advances in the erosion cycle, but that the end product, namely the open valley, wider than the meander belt and with meander-scalloped sides, is the same in all cases.

New terms might, perhaps with advantage, be invented to designate the different stages in this evolution, or it might, perhaps, be wise to apply the familiar, though somewhat overworked terms, "youthful," "mature," and "old." If this were done "youth" might be made to cover the period up to the time when active down-cutting ceases; "maturity" the period from the latter until wide, meander-scalloped flood plains develop and until the spurs of incised meanders have been cleared away by sweep, and "old age" the remainder of the cycle.

Significance of uplift.—In the preceding discussion we have assumed rapid or slow uplift of the basin of a master stream flowing directly to the sea. We concluded that rapid uplift should result in one type of valley form; slow uplift in another. If this proposition holds true, we have a valuable criterion for determining from the form of the river valleys or thalwegs the rate of the uplift to

[1] C. F. Marbut, *Physical Features of Missouri*, p. 104; "Meanders," *Missouri Geological Survey Bull.*, X, 94–109. This is a very excellent discussion of meanders; both the flood plain and incised types.

which a region may have been subjected. In applying this criterion in practice, however, we immediately encounter the difficulty that not all regions have master streams flowing directly to the sea. The region we are studying may be located in the interior of a continent and on a relatively small stream. In such cases, can we use the form of the valley in arriving at conclusions as to the rate of uplift of the region? Obviously not without qualification.

In order to put this criterion on a basis where it may be applied in practice it is only necessary to bear in mind that the equivalent of *uplift* as used in this connection, is *any condition which will cause a stream or any section of a stream to behave as it would if it were independent and subjected to uplift.* In other words the equivalent of uplift may be thought of as any condition which lowers the local base level of a stream. Thus a lowering of the lake level in the case of a stream flowing into a lake or the deepening of a master stream, in the case of a tributary, is equivalent to uplift in an independent stream.

VARYING TYPES OF VALLEY FORM IN DIFFERENT PARTS OF A SINGLE DRAINAGE SYSTEM

Turning our attention once more to our hypothetical drainage system uplifted after having reached the old-age stage of development, we may readily see that, even in this simple case, the same type of valley will not, as a rule, develop in all parts of a drainage system. Take, for example, the case of rapid uplift. We have seen that the master stream would trench rapidly and would form an open valley of type 1 or, where the original course was meandering, the intrenched meander valley of type 2. What would happen meanwhile in the distant headwater branches?

Let the profile of the stream before uplift be represented by the line *a* of Fig. 10. The final profile of equilibrium after uplift may be represented by the line *c*. The trenching resulting from uplift would proceed upstream in such a way that at some time between uplift and final grading the profile would take a form somewhat like the line *b*. Now at a point *W* in its lower course the master stream, at the time represented by the profile *b*, would have cut down a distance represented by the length of the line *V*-*W*. At another

point, *Y*, farther up the valley the stream would, in the same time, have cut down a much shorter distance, represented by the line *X-Y*. The effect at the latter point, particularly on tributaries entering there, would be equivalent to that of a *slow* uplift. We should expect, therefore, to find valleys normal for such an uplift—namely, in-grown meander valleys—developing at that point.

Thus in the lower course of the master stream, valleys typical of a rapid uplift would be found, while in its upper or middle course, and in many of the tributaries, valleys typical of a slow uplift would be encountered.[1]

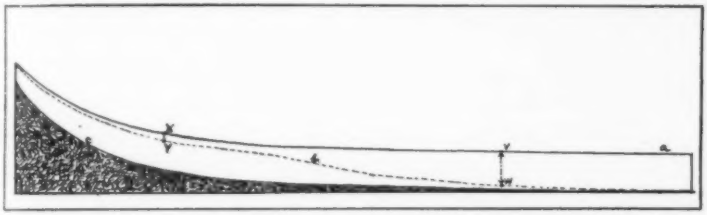

FIG. 10—Diagram to illustrate the explanation of the different types of valleys in different parts of a drainage system. Vertical scale greatly exaggerated.

Valleys of the *in-grown* type are especially likely to develop on the upstream side of resistant rock masses. These resistant masses retard the progress of a wave of trenching following an uplift and the stream above them, which must wait upon their down-cutting, is affected as by a *slow* uplift.

It follows, from considerations outlined above, that, for the *direct* determination of the rate of uplift by means of the form of the stream valleys, only the master streams, or those emptying directly into the sea may be successfully used. In other cases the character of the rocks and the distance from the sea may play a more important part than the rate of uplift. This fact does not, however, destroy the usefulness of the criterion, for it still indicates the rate of lowering of the *local* base-level of any particular stream in question.

[1] A. Penck, in *Morphologie der Erdoberfläche*, Buch II, S. 348, calls attention to the tendency to the formation of meanders in the middle course of a stream; also to the fact that meanders are characteristic of an underloaded stream when it is so placed that it cannot work downward.

Effects of a tilting uplift.—In the above analysis we have considered the drainage system as being uplifted bodily—the mouth of the stream as much as its headwaters. We were led to the conclusion that under those conditions the type of valley in the upper course of a stream would normally be different from that in the lower course.

If the uplift were of a tilting nature with the axis at the mouth of the master stream and with the headwaters receiving the greatest uplift, the upper reaches of the stream would feel the effects of the uplift as soon as any other part of its course, and, if the uplift were rapid, it would seem that the type of valley associated with rapid uplift would prevail throughout.

If we may safely reason from the deductions outlined above, we may conclude that, where rock textures do not complicate matters too much, a drainage system with valleys of the type representing rapid uplift in its lower part, and with types representing slow uplift in its upper branches indicates, though, on account of other complicating factors, it may not prove, a rapid bodily uplift of both headwaters and lower course, while one displaying the forms associated with rapid uplift throughout its course indicates a rapid *tilting* uplift, with maximum elevation in the upper parts of the drainage system. Effects of tilting in the opposite direction need not be considered here: the reader may readily work them out for himself.

If this criterion of tilting is found to hold good it might be possible, in the case of a fairly extensive dendritic drainage system, to determine the axis of tilting by noting the types of valleys in streams coming in from different directions and therefore affected differently by the tilting.

SPECIFIC APPLICATION OF THE CRITERIA.

As an example of the application of the criteria outlined in preceding paragraphs we may take the case of the Mississippi River as illustrated on the Waukon (Iowa-Wis.) sheet. The river here is flowing in a flat-bottomed valley of the open type some two miles wide and with steep, smoothly trimmed and parallel

valley walls some 400 feet in height. The river swings from side to side of the valley in broad, open curves which could scarcely be dignified by the name of meanders.

According to our deductions, an open valley of this type indicates a great predominance of *sweep* over the other processes at work in the formation of the valley. Such a valley might, as we have seen, be brought about by either one of two conditions: (*a*) a rapid deepening of the valley to grade, with subsequent widening, or (*b*) a deepening so slow—perhaps on account of a heavy sediment load—that sweep predominated from the beginning, continually shifting the locus of lateral cutting and thereby widening the valley uniformly as down-cutting proceeded.

In the valley of the Mississippi itself there appears to be nothing to indicate which of these is the correct interpretation, but when the tributary valleys are examined a clue presents itself. They are found (so far as can be judged from the contour maps, which are not so clear as could be desired) to enter through incised meander valleys, most of the larger of them clearly of the *intrenched* type. This form of valley indicates that the local base-level was lowered quickly (the equivalent of rapid uplift), and that the first of the two possible explanations mentioned is more probably the correct one. Such a conclusion should be tested further by the examination of other tributary valleys entering the Mississippi above and below this point. If all the larger tributaries are found to agree in their testimony, the hypothesis of rapid down-cutting by the main stream is strengthened.

The possibility that the valley of the Mississippi, at the point referred to, may, at one time, have carried a glacial stream much larger than the present river, and the fact that the valley is now silted up to a depth of from one to two hundred feet must be taken into consideration, but it does not seem to me to alter essentially the interpretation. Glacial waters may, however, be partly responsible for the remarkable sharpness of the features of the valley walls. In all essential particulars, aside from this sharpness, the valley is similar to that of the Kanawha at Charleston (Fig. 1).

SUMMARY

A study of the form of the immediate valley or thalweg of a large number of streams leads to the recognition of three distinct valley types. These are: (1) the open, comparatively straight and straight-walled valley through which the stream swings in more or less open curves, but in which the curves of the stream do not, except in the very earliest stages, correspond, necessarily, with the curves of the valley as a whole; (2) that form of the incised meander which we may call the intrenched meander, in which the meandering stream has sunk itself with little modification into bed-rock. (In this type undercut bluffs and slip-off slopes are not well developed, though they may be present to some extent. The greater part of the land within the loops retains the original height of the upland); (3) the form of the incised meander which we may designate the *in-grown* meander from the fact that as it sinks itself into the rock it is continually growing so that its final form and size may be very different from that at the beginning of incision. This type of incised meander does not necessitate a particularly meandering course of the stream in the cycle preceding the incision—the meandering course may *develop* as down-cutting proceeds. The valley is characterized by marked undercut bluffs and slip-off slopes. The evidences of growth or expansion during incision are very clearly expressed in the form of the valley.

A valley belonging to any one of these types exhibits a series of characteristic valley forms as it advances in the erosion cycle from the youthful to the mature and old-age stages of development, but there is a strong tendency for the final stages of all the types to become alike.

A large factor in determining the form of the valley, it seems, is the relative rate of down-valley sweep of the river curves. Only when this sweep is subordinate to down-cutting may any form of the incised meander valley be developed.

In a large drainage system rapidly and bodily uplifted, the master streams will normally develop valleys or thalwegs of type 1, the open valley, while in many of the headwater branches valleys of type 3, the in-grown meander, may be the common form.

Any barrier of hard rock in the course of a stream may, by retarding down-cutting, allow the development of valleys of type 3 above the barrier at the same time that below it type 1 is the prevailing form.

In conclusion, it appears that it is the ratio of the rate of vertical down-cutting to that of lateral cutting and sweep which determines the form of the thalwegs. When down-cutting predominates, valleys of types 1 or 2 are formed; when sweep is dominant the open valley, type 1, is the result. It is only when conditions lead to continuous down-cutting so slow that it is equaled or exceeded by lateral cutting, yet rapid enough to make sweep subordinate, that the in-grown meander type of valley may be produced.

These conclusions, if correct, establish valuable criteria for the interpretation of the physical history of a region from the form of its valleys.

STREAM PIRACY AND NATURAL BRIDGES IN THE LOESS OF SOUTHEAST MISSOURI

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Missouri School of Mines

In Perry and Cape Girardeau counties, in southeastern Missouri, are loess deposits locally deeply gullied into a sort of rather mild badland topography. During the field season of 1913, the writer observed in this gullied loess an unusual combination of erosion phenomena, which illustrates in miniature one process of formation of natural bridges.[1]

Along a roadside a gully had cut into the loess to a depth of over six feet, and, owing to a filling of bowlders and débris at its lower end, had established a temporary base level. The tributary gullies were also near this base level, and during a recent storm had actively widened their valleys.

At a distance of perhaps ten feet from the junction of the main gully and a tributary, undercutting of the banks in opposing directions had reduced the divide at that point to a wall of loess not more than fifteen inches thick, the upper grassy portion of which had crumbled away, lowering the divide a foot or two, at the narrowest place, and forming a sort of saddle.

The bed of the larger ditch was ten or twelve inches lower than that of its neighbor, and, in continually wearing against the loess wall, had completely undercut the divide, opening a gap about a foot high by eighteen inches long, leaving a natural bridge of loess, with an open arch between the two gullies.

The further undercutting of the larger channel had so encroached upon its smaller and higher neighbor as completely to divert the drainage of the latter through the arch, over a low rapids, leaving the lower course of the beheaded gully several inches higher than its former upper course, with a distinct step between them. The

[1] H. F. Cleland, "North American Natural Bridges, with a Discussion of Their Origin," *Bull. Geol. Soc. America*, XXI (1910), 313–38.

increased gradient of the diverted portion had caused it to intrench itself slightly in the bed of its old course.

The case illustrates both stream piracy and the formation of natural bridges by lateral erosion. The writer observed two other smaller bridges formed in the same way. In the two latter cases, however, drainage was not diverted. Several cases were noted in which the undercutting was nearly complete.

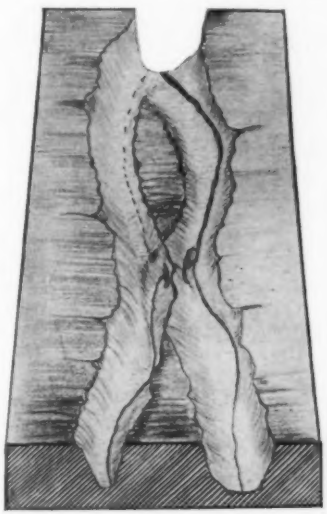

FIG. 1

At the time the writer observed the foregoing features all the gullies were dry. One living in a region of loess or badland formations subject to rapid local erosion could doubtless study these features in actual process of formation during any heavy shower.

Fig. 1 is a rough sketch, showing the general features described in the foregoing material.

CRYSTALLOBLASTIC ORDER AND MINERAL DEVELOPMENT IN METAMORPHISM

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VARIATIONS IN DEGREE OF METAMORPHISM

Metamorphic rocks often exhibit, within the same region, considerable variation in the kind and especially in the amount of metamorphism to which they have been subjected. This is particularly true of intrusive contact zones where a more or less uniform decrease in metamorphism may be observed away from the intrusive body.[1] In regional metamorphism, too, gradations may be found in the degree of alteration of the crystalline schists; but the variations here are much less regular than in the former case.

Recognizing the fact that long duration of time is an essential requisite for the course of metamorphism,[2] we may assume that a metamorphic rock, whatever may have been the forces concerned in its origin, must have passed through a sequence of stages of

[1] See A. Geikie, *Textbook of Geology*, 4th ed. (1903), p. 773; G. Barrow, "On an Intrusion of Muscovite-biotite Gneiss in the Southeastern Highlands of Scotland, and Its Accompanying Metamorphism," *Quar. Jour. Geol. Soc. Lond.*, XLIX (1893), 330.

[2] Grubenmann lays emphasis on the great importance of time. He writes, "Die Temperatur darf wohl als der bedeutendste Faktor der Metamorphose betrachtet werden."—*Die Kristallinen Schiefer*, 2d ed. (1910), p. 51.

development leading up to its present form; and we may assume further that, in any given region, provided we consider only rocks which were once of similar nature, this sequence of stages is represented, in a general way, by a graded series from the least altered to the most altered specimens. Within the writer's personal observation the best area illustrating such variations, in the case of regional metamorphism, is the Narragansett Basin in southern Rhode Island.

STAGES OF METAMORPHISM IN SCHISTS OF THE NARRAGANSETT BASIN

The Narragansett Basin is a structural basin consisting of a downfolded and downfaulted block of Carboniferous mudstones, sandstones, and conglomerates of fresh water deposition. These strata were folded and were altered by dynamic and static metamorphism during the Appalachian Revolution. They appear to have been near enough alike in their original state to justify a comparison of them now.

The characters of these rocks, as investigated by the writer,[1] led him to group them, according to their degree of metamorphism, in four stages, designated A, B, C, and D. "The metamorphism is incipient in Stage A; distinct, but rather low, in Stage B; considerable to high in Stage C; and at a maximum in Stage D."[2]

The criteria used in distinguishing between these stages were: amount of granulation or distortion of clastic components; deformation of pebbles, fossils, and such original structures as cross-bedding, ripple-mark, etc.; proportion of new or metamorphic minerals; proportion of recrystallized components; shape of mineral grains; mode of aggregation of constituents; degree of parallelism of minerals with unequal dimensions and of elongate mineral aggregates; perfection of rock cleavage (fracture); and gloss on fracture surfaces of the rock.

[1] F. H. Lahee, "Relations of the Degree of Metamorphism to Geological Structure and to Acid Igneous Intrusion in the Narragansett Basin, R.I.," *Am. Jour. Sci.*, (4), XXXIII, 249, 354, 447.

[2] *Ibid.*, p. 355.

Briefly the characters of the four stages may be summarized as follows:

Stage A:

Megascopic.—Fracture of rock irregular, or there may be a tendency to break parallel to the bedding. The fracture surfaces are dull. Original structures, fossils, and pebbles are not deformed.

Microscopic.—Nearly all the constituents are of clastic origin, with clastic outlines. Sericite, the first metamorphic mineral to appear, may be present in small quantity. There is no parallel alignment of elongate or flat minerals, unless parallel to the bedding, and then this arrangement is primary.

Stage B:

Megascopic.—There is a fair secondary cleavage. The cleavage surfaces have a faint gloss due to the parallel orientation of microscopic blades of sericite. Original structures, fossils, and pebbles may be somewhat distorted.

Microscopic.—Clastic grains are bent, strained, or crushed. Quartz grains may be granulated about their edges, or in bands that cross them, or entirely, and in the latter case the aggregates of grains may have been pressed out into lenticular form. Sericite is more abundant than in Stage A and a large proportion of it has parallel orientation. It is the principal cleavage-maker. Pebbles in conglomerate are thinly coated with this mica.

Stage C:

Megascopic.—The secondary cleavage is very good and the fracture surfaces have a brighter gloss. Fossils are usually unrecognizable. Pebbles may be much flattened or elongated. Original structures have been obscured or destroyed. Metacrysts[1] begin to be of marked importance.

Microscopic.—Much of the crushed clastic quartz has been recrystallized, the new grains being somewhat elongate parallel to the schistosity. Feldspar of secondary origin is occasionally seen. Sericite is very abundant and may show signs of growing coarser, to muscovite.[2] Its plates and laths lie approximately parallel. In the conglomerates sericite may be observed developed within the pebbles, but most other new minerals occur in the paste only. The mica coatings of the pebbles are thicker.

Stage D:

Megascopic.—A thin, perfect cleavage characterizes the rock. There may be a false cleavage. Fracture surfaces have a high sheen. Original structures are much deformed. Pebbles are either sheet-like or spindle-shaped, much longer than they are thick. No sign of fossils can be discovered.

[1] The name "metacrystals" was proposed by Lane for phenocrysts in metamorphic rocks, these crystals being of later origin than the groundmass (A. C. Lane, "Studies of the Grain of Igneous Intrusives," *Bull. Geol. Soc. Am.*, XIV [1903], 369).

[2] Grubenmann, *op. cit.*, p. 86.

Microscopic.—All the quartz is secondary. It often occurs in long thin blades and in thin plates parallel to the schistosity. Sericite has given place largely to muscovite. There is extreme parallelism of the constituents which are clear and free from indications of strain. The metamorphic (secondary) minerals are as abundant in the pebbles as in the paste. Recrystallization in pebbles and paste is at a maximum.

STUDY OF THE METACRYSTS

Many of the fine-grained schists of the last three stages contain metacrysts, or pseudophenocrysts, of ilmenite, biotite, garnet, and

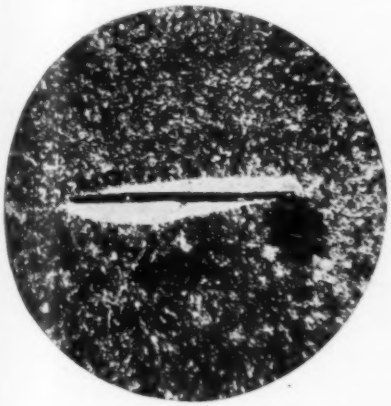

FIG. 1.—Crystal of ilmenite sandwiched between layers of secondary quartz. 15 diameters.

FIG. 2.—Metacrysts of biotite and ilmenite in a rock with poor flow cleavage. (See Fig. 3.) 15 diameters.

ottrelite, named in order of decreasing frequency. The same minerals are found in some of the coarser rocks, but in these they are no longer conspicuous for their relatively large size. The relations of these four minerals to one another and to the schistosity in the different stages of metamorphism will be described herein in greater detail than was possible in the earlier paper on this subject.[1]

Ilmenite.—In rocks of Stage B metacrysts of ilmenite occur as tabular crystals, thin or thick as the case may be (see Figs. 1, 2,

[1] F. H. Lahee, *op. cit.*

and 3).[1] When examined under the microscope they are quite opaque. Occasionally they contain small inclosures of quartz and sericite, the principal constituents of the groundmass, and their edges are more or less serrate where they have grown against these minerals. Since ilmenite is not a silicate, it evidently made room for itself by replacing the minerals of the groundmass and not by absorbing them. Its efficiency of replacement must be high for it contains relatively few inclusions.[2]

Where the groundmass has no parallel arrangement of its constituents and even where it does possess a flow cleavage that is not too well marked, the ilmenite plates lack definite orientation. Fig. 2 illustrates a portion of a rock slide in which the ilmenite crystals were scattered at random and the schistosity curved round them. It is evident here either (1) that they developed before the flow cleavage and that, in this case, the shearing of the rock was not sufficiently rapid or strong to bend or break them; or (2) that they grew *pari passu* with the schistosity, but that the shearing force was not enough to prevent their random orientation, that is, that their molecular forces were greater than the exterior stress. In either event they may have suffered some rotation.

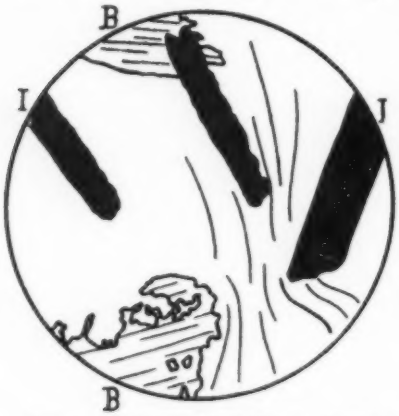

FIG. 3.—Outline sketch of Fig. 2. Ilmenite crystal partly inclosed in biotite. Note curving of the schistosity. I, ilmenite. B, biotite.

[1] The photomicrographs used for this article were taken by Professor E. C. Jeffrey of Harvard University. The writer is happy to express his deep gratitude to Professor Jeffrey for this compliment and for valuable suggestions in preparing the illustrations.

[2] Cf. Van Hise, "In proportion as minerals are unable to absorb, they are able to enclose."—Treatise on Metamorphism, *U.S.G.S. Monog.*, XLVII (1904), 700.

In many of the finer mud schists the plates of ilmenite are coated with quartz[1] (see Fig. 1). The correct explanation for this phenomenon is not certain. The suggestion has been made that the quartz-ilmenite aggregate occupies a space which was formerly an actual cavity or a potential one. This does not seem to be true, for the space has not the shape of a rent, and neither the ilmenite nor the quartz granules of the rim reveal any evidence of having grown inward from the walls of a cavity. The form of the present quartz-ilmenite aggregate indicates that it is a replacement of an earlier larger crystal of ilmenite or of some other flat mineral.

In rocks with a good flow cleavage, produced both by thin plates of recrystallized quartz and by sericite (Stages C and D), the ilmenite crystals are parallel to the schistosity (see Figs. 4 to 8). It is not at all likely that the ilmenite here originated before shearing, for rotation could not account for the nearly exact parallelism of all the plates in the rock. Two possibilities remain, then: that the building of its crystals was either contemporaneous with, or subsequent to, the origin of the schistosity. If the first condition was the actual one, the ilmenite as well as the quartz and sericite contributed to the accommodation of the rock to the stress, the accommodation being brought about by crystallization and recrystallization, i.e., by chemical processes. If, on the other hand, the second condition was the real one, the ilmenite plates did not contribute toward the accommodation, but acquired their parallelism on account of a property of the groundmass constituents (principally quartz and sericite) to dissolve more readily parallel to the schistosity than in any other direction. Now, it is known that

[1] Ilmenite has been described by Wolff and Pumpelly as bordered by chlorite; by Renard as bordered with sericite; and by Williams as coated by biotite. See the following:

A. Renard, "Recherches sur la composition et la structure des phyllades ardennais," *Bull. Mus. R. His. Nat. Belg.*, I, 212; II, 127; III, 84, 230 (1884).

G. H. Williams, "The Greenstone Schist Areas of the Menominee and Marquette Districts," *Michigan U.S.G.S., Bull. 61* (1890), 200.

J. E. Wolff, "On Some Occurrences of Ottrelite and Ilmenite Schists in New England," *Bull. Harv. Mus. Comp. Zoöl.*, XVI, 8 (1890), 162.

R. Pumpelly, J. E. Wolff, and T. N. Dale, "Geology of the Green Mountains in Massachusetts," *U.S.G.S. Monog.*, XXIII (1894), 183.

FIG. 4.—Ilmenite and biotite metacrysts in a rock having good flow cleavage. 15 diameters.

FIG. 5.—Outline sketch of Fig. 4. The ilmenite crystals lie parallel to the schistosity. One of them and the biotite contain inclusions of quartz. The irregularities of both ilmenite and biotite are due to growth of these minerals against the constituents of the groundmass.

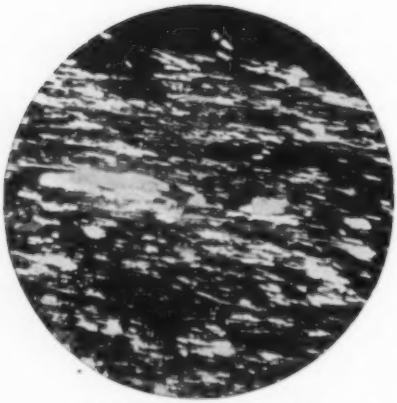

FIG. 6.—Photomicrograph of a rock belonging to "Stage D." The quartz is all secondary. The schistosity is well developed. An ilmenite lath with two quartz inclusions lies parallel to the schistosity. (See Fig. 7.) 10 diameters.

FIG. 7.—Outline sketch of Fig. 6. Shows position of the ilmenite crystal, and its relation to the flow cleavage.

the facility of solution and crystallization varies in different crystallographic directions and for different crystal forms of the same substance;[1] but, while this property might be suggested to explain the replacement of sericite, it could not account for the replacement of quartz, because quartz does not have crystallographic parallelism. Quartz plates would not tend to dissolve more readily at their edges than on their broad surfaces. If the rule applied to sericite only, and not to quartz, we should expect to find evidence of selective replacement by the ilmenite—substitution of sericite and not of quartz. However, no such evidence is apparent. The necessary conclusion is that the ilmenite plates, like the sericite and quartz, were formed during the operation of the shearing stress.[2]

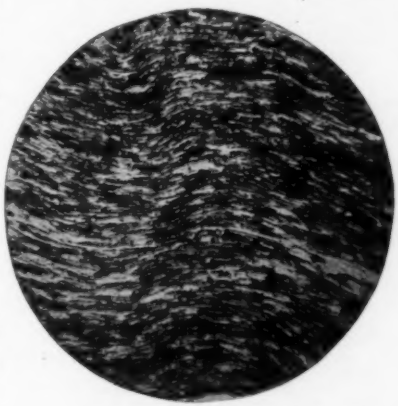

FIG. 8.—Similar to Fig. 6. An ilmenite lath may be seen near the top of the figure. The flow cleavage has been somewhat folded, thus showing a tendency toward the formation of a false cleavage. 20 diameters.

The relations of the growth of ilmenite to the development of the schistosity are brought out in Figs. 9 to 12. Figure 9 exhibits a garnet grain (about 1/16 inch in diameter) which partly inclosed an ilmenite crystal. The portion of the ilmenite outside the garnet is about half as thick as that inside and lies parallel to the schistosity which wraps round the garnet. The events illustrated by the photograph seem to have been as follows: (1) growth of a relatively thick plate of ilmenite; (2) inclosure of part of this plate by a crystal of garnet; (3) shearing of the rock, which produced, in the groundmass, a schistose structure that curves round the garnet, and

[1] See writings by Goldschmidt and others.

[2] Van Hise states that no minerals show crystallographic orientation under mass-static conditions (*op. cit.*, p. 689). When minerals are crystallographically, as well as dimensionally, oriented, the suggestion derived from Goldschmidt's work should receive careful consideration.

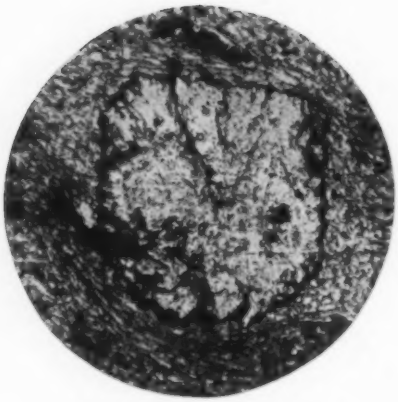

FIG. 9.—A large garnet grain partly inclosing an ilmenite lath which has been bent into parallelism with the schistosity. (See Fig. 10.) 15 diameters.

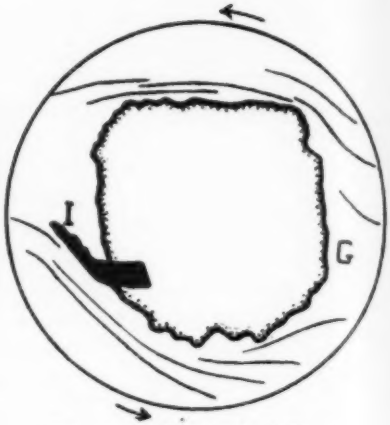

FIG. 10.—Outline sketch of Fig. 9. The schistosity, shown by curving lines, wraps round the garnet grain. The position and shape of these curves indicates that the garnet was rotated (arrows). G, garnet. I, ilmenite.

FIG. 11.—Ilmenite bent and inclosed in garnet. (See Fig. 12.) 25 diameters.

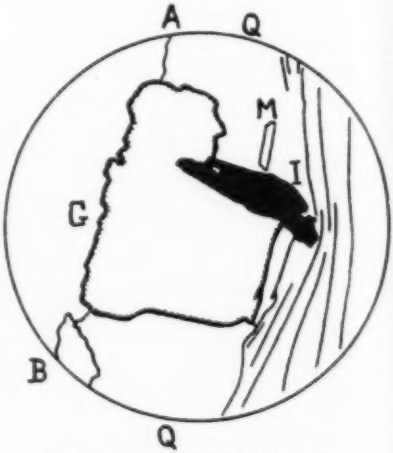

FIG. 12.—Outline sketch of Fig. 11. M, muscovite. Q, quartz. Between A and B is the edge of the rock section, under the cover glass.

resulted in the thinning of the protruded part of the ilmenite and in its being bent into parallelism with the adjacent schistosity. Since the ilmenite reveals no sign of fracture at its "elbow of deformation," its distortion was probably accomplished by gliding or by less regular molecular readjustment. In either case the facility of adjustment in the ilmenite must have been at least as great as the ease of accommodation of the groundmass; i.e., deformation of the ilmenite must have kept pace with development of the schistosity.

Fig. 11 shows a similar instance of an ilmenite crystal partly inclosed by garnet, but here the projecting portion of the ilmenite is less bent and less thinned than in the preceding example. A small re-entrant at the elbow of deformation, now filled with secondary quartz, suggests that the shearing produced a small fracture, subsequently perhaps more or less restored, or—and this is more probable—that it locally increased the tendency toward molecular readjustment at that place, making the ilmenite susceptible to replacement by quartz.[1]

Garnet.—Garnet metacrysts are found in rocks assigned to Stages C and D. In all cases there is clear evidence that this mineral originated before shearing entirely ceased.[2] Figs. 9 to 12 bring out this feature very well. The structure of the groundmass, particularly in Figs. 9 and 10, indicates rotation of the garnet crystal, and a similar relation appears in Figs. 13 and 14. Fig. 13 shows also the so-called "tails" of quartz, light areas that extend in opposite directions a short distance out from the metacryst, parallel to the schistosity. Such "tails" are without doubt in process of formation during the shearing of the rock. Consequently, their peculiar association with the garnet metacrysts is another fact pointing to the conclusion that the garnet was formed before the cessation of mechanical deformation.

Biotite.—As would naturally be expected, in rocks with no flow cleavage, biotite plates, if present, have no definite orientation.

[1] "The experimental work of Barus and Hambeuchen together has completely demonstrated that a state of strain in substances is favorable to chemical action."—Van Hise, *Treatise on Metamorphism*, p. 691.

[2] Cf. this statement by Leith: Garnet, staurolite, andalusite, etc., "in many if not in most cases crystallized out later than the principal cleavage-making minerals. . . ."—C. K. Leith, "Rock Cleavage," *U.S.G.S., Bull. 239* (1905), 93–94.

FIG. 13.—Garnet crystal in schist. (See Fig. 14.) 15 diameters.

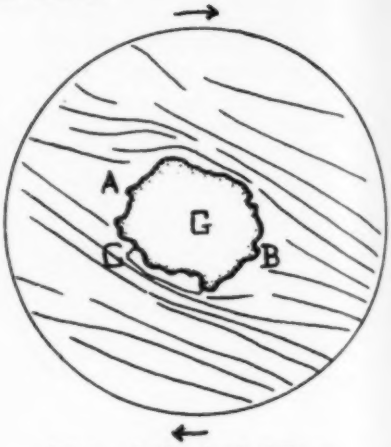

FIG. 14.—Outline sketch of Fig. 13. C, chlorite. A and B, the poles of minimum compression. The quartz "tails" extend out from the garnet (G) from these poles in opposite directions. The curves of the schistosity indicate rotation of the garnet (arrows).

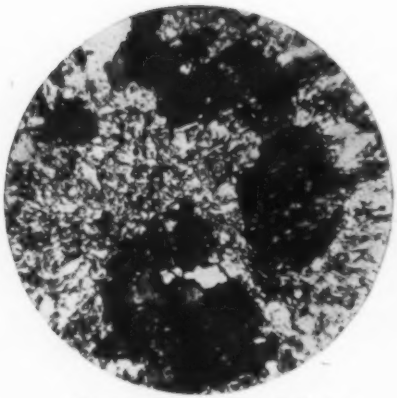

FIG. 15.—Metacrysts of biotite in a rock having no flow cleavage. The irregular outlines of the metacrysts are determined by the adjacent quartz grains. The biotite has neither dimensional nor crystallographic orientation. 12 diameters.

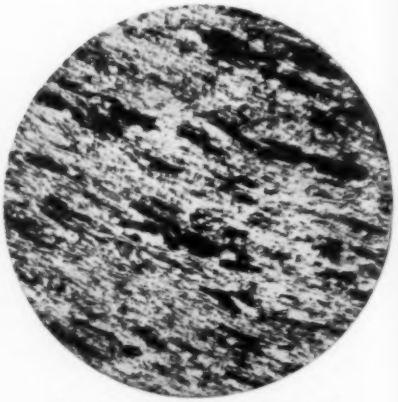

FIG. 16.—Photomicrograph of a rock with a well-developed flow cleavage. The biotite plates (dark) have dimensional parallelism and many of them have crystallographic parallelism. 10 diameters.

This is illustrated in Fig. 15. The biotite is here mottled lighter and darker. The light portions were once quartz grains of the groundmass, which were absorbed or replaced by the mica in its growth. The dark areas contain a large proportion of opaque carbonaceous matter which was also a part of the groundmass and which was included, but not absorbed nor replaced. For this reason the structure of the groundmass is nearly as distinct within these biotite crystals as it is outside. Their edges, like those of the ilmenite plates, are irregular because of their growth against constitutents of the groundmass, and the white inclusions and reentrants are unabsorbed quartz grains.

In schists belonging to Stage C, in which ilmenite has already acquired parallel orientation and much of the quartz of the groundmass bears evidence of recrystallization, biotite still lacks dimensional and crystallographic parallelism. The biotite crystal photographed in Fig. 4 was cut where a quartz band passes quite through it. Its cleavage is nearly perpendicular to the schistosity and its length is only very little greater than its width.

Under conditions of extreme metamorphism (Stage D), biotite acquires dimensional parallelism (Fig. 16). At this stage the quartz of the rock is of secondary origin and the sericite, the earliest new mineral to appear at the inception of metamorphism, has given place to muscovite. Crystals of these three minerals are roughly of the same size. The quartz and white mica have become relatively larger, and the biotite has become relatively smaller.

Several slides show that the biotite was subsequent to the ilmenite in respect to its origin. In Figs. 2 and 17 biotite crystals have partly inclosed adjacent plates of ilmenite. The projecting ends of the ilmenite in Fig. 17 (see also Fig. 18) have quartz borders like those illustrated in Fig. 1. This quartz border, once entirely surrounding the ilmenite, was absorbed or replaced by the biotite in just the same way as the quartz grains of the groundmass, as shown in Fig. 15. This is the explanation of the light halo that encircles the included portion of the ilmenite.[1]

[1] Professor Wolff describes a rock in which ilmenite plates are coated with chlorite except where they are included in ottrelite (J. E. Wolff, "On Some Occurrences of Ottrelite and Ilmenite Schist in New England, *Bull., Harv. Mus. Comp. Zoöl.*, XVI, 8 [1890], p. 162).

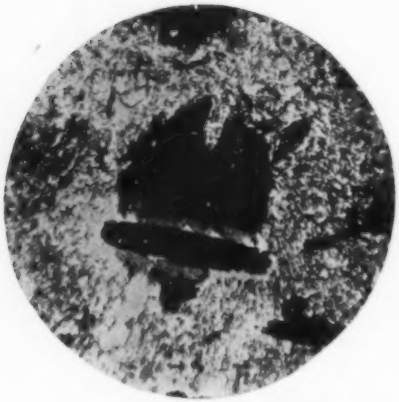

FIG. 17.—Ilmenite crystal nearly inclosed in biotite. (See Fig. 18.) 12 diameters.

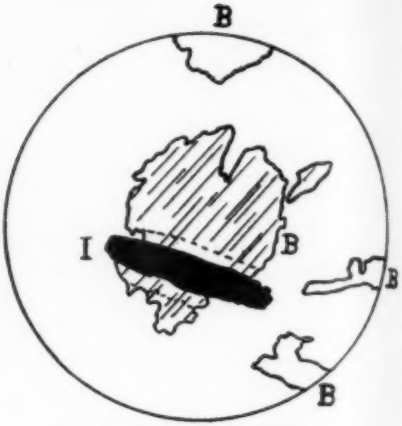

FIG. 18.—Outline sketch of Fig. 17. B, biotite. I, ilmenite. The dotted lines represent the edge of what used to be a quartz coating on the ilmenite. Cf. Fig. 1.

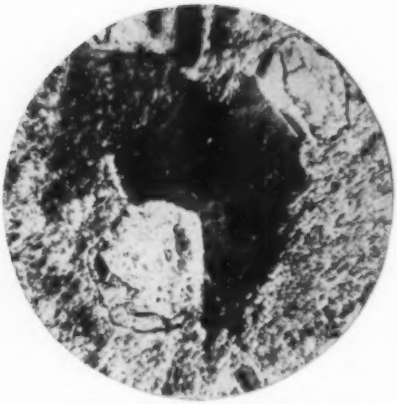

FIG. 19.—Biotite and garnet metacrysts. (See Fig. 20.) 25 diameters.

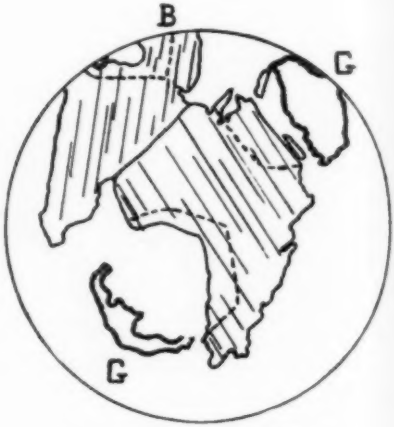

FIG. 20.—Outline sketch of Fig. 19. B, biotite. G, garnet. The dotted lines represent the edges of the garnet crystals before replacement by biotite.

Fig. 19 shows a large biotite plate which has replaced a portion of a garnet crystal. The hexagonal outline of the cross-section of the latter is plainly visible in the mica. Between the biotite and the outlying strip of garnet (Fig. 20) is a clear space occupied chiefly by secondary quartz and muscovite. Two or three minute grains of garnet in this space suggest that this mineral once existed as a complete crystal which was replaced partly by biotite and partly by the quartz-muscovite aggregate.[1] Obviously, biotite developed after garnet.

FIG. 21.—Metacryst of ottrelite showing hour-glass twin. 25 diameters.

Ottrelite.—Although ottrelite (Fig. 21) was seen only in rocks with a good flow cleavage, its crystals were never observed to have dimensional parallelism. Leith states, however, that it may occasionally show a definite orientation.[2] Plates of this mineral sometimes wholly or partly include metacrysts of ilmenite and biotite. Ottrelite, then, originates later than ilmenite, garnet, and biotite, and as a rule subsequent to the development of the schistosity.

SUMMARY AND CONCLUSIONS

The facts presented in the foregoing description of microscopic structures in the Narragansett Basin schists may be summarized as follows:

1. The commonest minerals in these rocks are sericite, muscovite, quartz, ilmenite, garnet, biotite, and ottrelite. Quartz and sericite or muscovite form a large percentage of the composition of

[1] Cf. B. K. Emerson on "skeleton crystals" of garnet: "Note on Corundum and a Graphite Essonite from Barkhamsted, Connecticut," *Am. Jour. Sci.* (4), XIV (1902), p. 234.

[2] "Rock Cleavage," *U.S.G.S., Bull. 239* (1905), p. 44.

each rock. They may occur as a relatively fine groundmass in which ilmenite, biotite, garnet, and ottrelite may be present as metacrysts.

2. Minerals that crystallize or recrystallize with parallel orientation promote the accommodation of the rock to the stress.[1]

3. A mineral that is shown to acquire parallel orientation at an *early* stage of dynamic metamorphism will grow with parallel orientation in later stages, provided shearing continues.

4. A mineral that acquires parallel arrangement at a *late* stage of dynamic metamorphism may develop at an earlier stage, but in this case it originates after shearing has ceased and then has no definite orientation.

5. In the Narragansett Basin schists sericite is the first metamorphic mineral to appear and is the first to acquire definite orientation of its crystals under conditions of stress. In Stage D it may give place to muscovite.

6. Quartz is chiefly clastic in the early stages of metamorphism and secondary (recrystallized) in the later stages. In the accommodation of the rock to stress it assists first by granulation and later by recrystallization. It acquires dimensional, but not crystallographic, parallelism in Stages C and D.

7. The order in which the minerals acquire both dimensional and crystallographic parallelism, beginning with the earliest, is as follows: sericite, ilmenite, biotite, ottrelite. Secondary quartz, having only dimensional parallelism, would come between ilmenite and biotite.[2]

8. The order of origin of the metacrysts, as shown by their relations to one another, is: ilmenite (first), garnet, biotite, and ottrelite. Grubenmann calls this a crystalloblastic order (Reihe).[3] He gives the following succession for minerals of metamorphic origin: titanite, rutile, hematite, ilmenite, garnet, tourmaline,

[1] Cf. Leith: "Minerals showing the best evidence of recrystallization are those best adapted by their shape and dimensions to conditions of unequal pressure," *op. cit.*, p. 95.

[2] Leith places quartz after the micas in respect to its cleavage-making capacity. Commencing with the best cleavage-maker, his order is: micas, hornblende, quartz, and feldspar (*op. cit.*, p. 64).

[3] *Die kristallinen Schiefer*, p. 91.

staurolite, cyanite—epidote, zoisite—pyroxene, hornblende—magnesite, dolomite, albite, mica, chlorite, talc—calcite—quartz, plagioclase—orthoclase, microcline. In general the series is one of decreasing specific gravity and of increasing molecular volume.[1] It is interesting to compare Grubenmann's values for specific gravity and molecular volume[2] for the four minerals under dicussion. They are:

	Sp. gr.	Mol. vol.
Ilmenite	4.70	31.7
Garnet (almandite)	4.11	119.8
Biotite	3.06	152.2
Ottrelite (chloritoid)	3.50	69.6

Ottrelite is an exception to Grubenmann's generalization.

Mention may be made here of other references to crystalloblastic order. Wolff[3] has shown that ottrelite may crystallize out before ilmenite. According to F. W. Clarke,[4] andalusite, sillimanite, and kyanite originate in the order named, with increasing metamorphism. This has been recorded by several observers. Van Hise[5] states that garnet represents a less advanced stage of alteration than staurolite. Where both of these minerals occur in the same schist garnet is frequently inclosed in staurolite. The present writer has seen this in many of the New Hampshire schists. Leith[6] writes: "We know definitely that quartz generally crystallizes before feldspar, and mica and hornblende before quartz and feldspar. . . . . Muscovite and biotite, when they occur together, usually develop simultaneously. . . . . Exceptionally the muscovite evidently crystallizes before the biotite." Garnet and staurolite are listed by Leith with the minerals that crystallize out after the cleavage-making minerals.

October 18, 1913

[1] *Op. cit.*, pp. 91–92. [2] *Op. cit.*, pp. 54–55.

[3] J. E. Wolff, "On some Occurrences of Ottrelite and Ilmenite Schist in New England," *Bull. Harv. Mus. Comp. Zoöl.*, XVI, 8 (1890), p. 163.

[4] "Data of Geochemistry," *U.S.G.S., Bull. 330* (1908), p. 528.

[5] C. R. Van Hise, *Treatise on Metamorphism*, p. 903.

[6] *Rock Cleavage*, pp. 93–94.

DIASTROPHISM AND THE FORMATIVE PROCESSES. VIII

THE QUANTITATIVE ELEMENT IN CIRCUM-CONTINENTAL GROWTH

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University of Chicago

In a previous article of this series (VI, p. 271) we endeavored to show that the depth of the ocean does not, in itself, set limits to the thickness of the strata laid down in it—when these are measured in the usual way—though it does determine the amount of material required to build the continental terraces oceanward to given distances and thus indirectly affects the thickness of strata actually attained by a given amount of material. As a quantitative factor, then, ocean depth has its importance in determining the amount of peripheral growth that may arise from a given amount of sediment; or, if the peripheral growth be given, the amount of sediment required to accomplish it and, by interpretation, the time. The unit of such quantitative measurement is found in terms of radial extension rather than stratigraphic thickness. This distinction of terms and methods arises from the fact that the ocean depth affects the *area* of each stratum because of its inclined attitude. The quantity of sediment in the circum-continental terraces is one of the elements by which the denudation of tributary land is measured; or, the rate of denudation being roughly known or assumed, the amount of sediment in the terraces is a measure of the time occupied in the formation of the terraces. It thus becomes not only an index of the antiquity of the terraces but gives some incidental testimony as to their history.

In such rough approximations as alone are possible in studies of secular denudation and accumulation, we may assume that the surface movements of the ocean waters were effective in the same way and to the same extent in all ages, though this assumption was

probably not strictly true and perhaps not approximately true in some cases. On this assumption the topset deposits of the continental shelves were of the same order of vertical depth as those recently formed. These we commonly agree to limit to the work of the upper 100 fathoms of water, though the sum of the thicknesses of the topset *strata* is not limited to 100 fathoms. The variable effects due to variable ocean depths are hence manifested almost solely in the foreset beds formed on the terrace faces overlooking the abysmal deeps.

Now if we assume, for a concrete example, that the slope of these foreset beds is 1 in 12, and that the mean depth of the abysmal part of the ocean adjoining is 5,000 meters, a symmetrical stratum of such an order as to advance the terrace horizontally one meter oceanward will be 1/12 meter thick, and will have a radial width of 60 kilometers. This stratum is equivalent to a vertical stratum one meter wide and 5,000 meters deep. If any other slope than 1 to 12 be taken as representative, a similar result follows; that is, a stratum that measures one meter horizontally at the surface, may be reduced to a vertical stratum one meter wide and 5,000 meters deep. The measure of *horizontal* advance and the measure of ocean depth thus appear to be the true factors in measuring sedimentary growth about the continents and to be conveniently simple in use. The *slopes* and the *stratigraphic* thicknesses may in many cases be neglected. They must of course be duly taken into account in their own appropriate fields but these are not precisely this field.

Notwithstanding the simplicity of the method thus deduced, all determinations of the quantitative values of the sediments of the circum-continental terraces are embarrassed by several serious difficulties. In spite of these, however, some working concepts may be formed that have value, even though they do not rise above very rough approximations.

In any specific attempt to deal with the sediments of the terrace shelf it is of course necessary to select the epoch or the formation from which the beginning of terrace growth is to be reckoned. It is idle in the present state of knowledge to go back of the Cambrian period and in many problems a much later date is best taken as the starting-point.

When this datum point is chosen there arise at once special difficulties in determining what were the landward and bottom boundaries at the start, for even the landward edge at the surface is largely concealed by later deposits, or has been cut away, while its lower parts and the bottom are generally inaccessible except as diastrophism has forced them into view. In few cases—probably in no case—has a terrace shelf throughout its history been built oceanward solely. In most cases the sea has advanced upon the land and the topset deposits have crept out upon the continent as well as oceanward. The landward advance has in many cases been interrupted by diastrophic movements and the shore line forced seaward, sometimes even beyond the initial line. Later a new advance has taken place from this line of retreat and so the oscillation has gone on. As a result of this the innermost edge of sedimentary deposits is liable to lie farther landward than the line from which growth began at the opening of the epoch chosen. The border sediments that thus lap upon the continent—defined as it stood at the outset—must of course be taken into account but not as extensions of the continent; they are rather superficial replacements or over-placements and, so far as determinable, are usually limited in depth and relatively small in quantitative value.

When pre-Paleozoic formations come out to the present coastline the location of the inner border of the terrace formed after the initial epoch is little better than conjectural. Some suggestions of value may spring from the trend of the border line in adjacent regions and from the consistency of value of any assigned configuration.

Probably the best approximation to the initial line of growth may be reached by setting aside for compensation purposes all sediments that are known to lap upon the continent as it stood at the outset, while such border sediments as are known to be deep and to show no signs of overlap are regarded as belonging to the continental outgrowth. In this it is assumed that any overestimates in the volume of the basal parts will be made good by the sediments of the overlap set aside for this purpose. It will be shown presently why the basal parts are not as likely to lead to overestimates as their apparent uncertainty may lead one to think. They are not wholly uncertain.

Where notable folding or faulting has taken place—and this is common near the coasts—it may be possible to determine more directly how far landward thicknesses of sediment of the order of the ocean depth have once extended, for the upturned beds may reveal them. Such observations must, however, usually give only a minimum extension; the evidences of the maximum extension have usually been cut away by erosion. An additional minimizing effect usually arises from the compression and distortion of the sediments under diastrophic thrusts. Their observed horizontal breadth may be much short of their original depositional breadth.

Students of terrestrial dynamics of all schools will probably agree that the borders of the continents have been the seats of exceptional folding and faulting. Observation and most theories, however diverse otherwise, lend support to this view. It is probably safe, and even ultra-conservative, to take the innermost line of continuous, thick, folded sediments as the landward border of the outward-built terrace, if, of course, these sediments are later than the time fixed upon as the beginning of the terrace-building epoch.

As already remarked, it is useless at present to consider circumcontinental terraces older than the Paleozoic, for the metamorphisms and distortions of the earlier terranes and their wide concealment forbid their treatment in any satisfactory way. None the less, there is no good ground to doubt that the oceanward borders of the Proterozoic lands were affected for long periods by the process of terrace-building. The clastics form a notable factor in the Proterozoic terranes; they form a factor, though a less notable one, of the Archean also, and, under the planetesimal view, of terranes below the visible Archean. Whatever, therefore, may have been the original oceanward slopes of the submerged borders of the continental nuclei during the very earliest eras, normal topset and foreset slopes encircling these should have been acquired during the progress of the Proterozoic era. Except, therefore, as modified by diastrophism late in Proterozoic time or at its close, the oceanward face of the continental masses should have borne the normal sedimentary configurations.

Regarding the effects of diastrophism, I think it will be agreed quite generally that the mean results of long stretches of time tend

rather to steepen the seaward gradients of the coasts than to flatten them; tend rather also to deepen the adjacent ocean than to shallow it. Theoretically this would seem quite certainly true if any tenable form of isostasy is effectual in diastrophism, and of this there seems less ground for doubt as inquiry goes on. On the observational side, the frequency of fore-deeps and of notable depressions off the continental edge not definitely shaped as fore-deeps seems to support this. It may be assumed, therefore, with much probability, that from Cambrian time onward topset and foreset action kept rebuilding the form of the circum-continental terraces into slopes that lay within their own normal range of variation—diastrophism aside—and that the mean ocean depth bordering the continents was not less than the normal mean depth. It may further be assumed with high probability that the mean effect of diastrophism, when it intervened, was to increase the border slope and the border depth, while it compacted, folded, and shortened the previous terrace outgrowth.

It appears, therefore, that if the upper edge of the landward border of the oceanic basin can be fixed approximately at the epoch from which the terrace growth is to be estimated, the slope of the terrace front and the depth of the adjacent ocean may be assumed to be of the present order, with some likelihood that this assumption is in reality conservative. These considerations relieve very appreciably the embarrassments that would otherwise affect the lower and landward configurations of the circum-continental terraces when we attempt to restore them, by interpretation, for any particular post-Proterozoic epoch.

The surface area of the present continental shelf between the shore line and the 100-fathom line is usually taken at 10,000,000 square miles, following Murray. The additions to be made to this to give the full area of the built terrace when its true border lies on the landward side of the present shore line, and the subtractions that are to be made when it lies on the seaward side, can only be roughly guessed at until the geological determinations of the coastal terranes of the several continents have reached a more advanced state, for any close estimate requires data not now available. None the less, it is worth while to make rough guesses of their values on

such data as we have, if only to see what would be their meaning if the facts were as they are guessed to be. Without any pretention to accuracy, permit me to assume, on the basis of rough inspections, that for the total post-Proterozoic outgrowth the landward additions to be made to the present shelf are of the order of 40 per cent of its area, and that the subtractions are of the order of 20 per cent. My real judgment inclines to make the former more nearly 50 per cent and the latter more nearly 10 per cent; but, proceeding on what seems to me a conservative assumption, the total area of outgrowth of the post-Proterozoic circum-continental terrace, neglecting the crumpling, compacting, and shortening that have arisen at times from lateral thrusts, amounts to 12,000,000 square miles.

The coast line of the present continent is sometimes taken as 125,000 miles. For our present purposes a measurement on less curved lines such as to represent the middle of the shelf belt encircling the continents is more suitable, and an estimate on this basis gives about 100,000 miles. The mean breadth of the constructive terrace will then stand at 120 miles. Checked by direct measurements in some of the best determined representative cases this seems conservative.

Let us now turn to the sources of sedimentary supply. It would lead to large errors if each unit of the sea border were assumed to be fed with sediment from the whole area of a strip of land abutting on it and reaching back to the heart of the continent, for in the first place, much of the drainage runs away from the coast, and, in the second place, there is more or less coastwise drift that denudes some tracts and builds out others disproportionately. These factors have no doubt varied from age to age, and perhaps varied greatly, but our immediate purpose is to secure a rough concept of processes as they now are and as they are related to the present accumulation of sediments. These may be qualified later to fit the mean secular conditions or the specific conditions of any particular problem.

At present the continental surfaces may be divided into three classes in respect to the transportation and lodgment of sediments: (1) transportation into interior undrained basins which contribute nothing immediately to circum-continental growth; (2) trans-

portation toward the continental interiors, with a possibility or probability that more or less of the sediment will at length reach the coast by a long and circuitous route but with a liability also of lodgment in lake basins, on low plains, in estuaries, embayments, gulfs, or other dependencies of the ocean, in consequence of which the sediments are consumed in building up depressions in the interiors of continents rather than building out their borders; and (3) direct transportation to the ocean down the border slopes of the continents; or in briefer terms: (1) internal drainage with no oceanic connection, (2) internal drainage with indirect oceanic connection, and (3) direct oceanic drainage. Murray places the area of the first class at 11.5 million square miles out of a total of 55.7 million, or a little more than 20 per cent. Leaving out the circumpolar region that cannot now be treated, there remain about 40 million square miles to be placed under classes two and three. I know of no authoritative division of this area between these classes, nor do I see how any but a somewhat arbitrary division can be made, for the two classes grade into one another in a very intricate way. If we put into class three only the drainage from the coastward sides of the mountains and other elevated tracts that so generally border the continents and add the drainage from the coastal flats where elevations are absent, the resulting area will be inferior to that of the great interior basins. So likewise if, for another line of approach, we turn to the geological record, as now known, the amount of clastic sediments embraced in the coastal slopes seems much inferior to that which is embraced in the great interior terranes. But, as a portion of the sediment that goes inward at first, later reaches the exterior of the continent, let the 40 million square miles be divided equally between the class which contributes to building up the interior and the class that contributes to building the continents outward. We have already taken the length of the circum-continental shelf belt as 100,000 miles. The assigned 20 million square miles tributary to it gives a working mean of 200 square miles tributary to every mile of length of the shelf.

Much study has been given to the determination of the average rate of denudation of land surfaces under present conditions in the

endeavor to establish a mean rate serviceable for computing the age of the earth. The more recent representative treatments dealing with specific data may be found in the papers of Walcott,[1] Sollas,[2] Dole and Stabler,[3] Joly,[4] Becker,[5] Clarke,[6] Arthur Holmes,[7] in which may also be found references to earlier studies. The data of the United States Geological Survey, as organized and tabulated by Dole and Stabler, and discussed by Clarke, furnish the most definite and best determined material available, as well as that most suited to our purpose. It is not the rate of the complete denudation of the land which we wish to use in this discussion but only that part of it which found lodgment on the continental terraces. This embraced chiefly materials of certain degrees of coarseness or gravity and a certain portion of the dissolved material.

Clarke has pointed out the wide differences in the data obtained from different regions and has assigned reasons for these. It appears in particular that the average denudation of the basins of the Amazon and Uruguay rivers is but a trifle over one-half that of the Mississippi and St. Lawrence basins, though the precipitation on the tropical basins is nearly double that on the temperate basins. The difference is assigned to the forest clothing of the tropical basins. It appears from such data as Clarke found available that the mean denudation of the river basins of Europe is 100 tons per square mile, of Asia 84 tons, of North America 79 tons, of South

[1] C. D. Walcott, "Geologic Time, as Indicated by the Sedimentary Rocks of North America," *Jour. Geol.*, I (1893), 639–76.

[2] Sollas, Brit. Assoc. Rept., Address to Sect. C, 1900, quoted by Joly in *Radioactivity and Geology* (1909), p. 246; Presidential Address, *Quar. Jour. Geol. Soc.*, May, 1909.

[3] R. B. Dole and H. Stabler, "Denudation," *U.S. Water Supply Paper 234* (1909), pp. 78–93.

[4] J. Joly, *Radioactivity and Geology* (1909), chap. xi; *Phil. Mag.*, 6th ser., XXII (1911), 358; *Trans. R. S. Dublin*, VII (1899), 23.

[5] George F. Becker, "The Age of the Earth," *Smith. Misc. Coll.*, LVI, No. 6 (1910). "Halley on the Age of the Ocean," *Science*, N.S., XXXI (1910), No. 795, pp. 459–61.

[6] F. W. Clarke, "A Preliminary Study of Chemical Denudation," *Smith Misc. Coll.*, LVI, No. 5 (1910); "The Data of Geochemistry," *U.S. Geol. Surv. Bull. 491*. 2d ed. (1911), p. 60 f., 137–42, 466–67.

[7] Arthur Holmes, *The Age of the Earth* (1913), chaps. iv–vi.

America 50 tons, and of Africa 44 tons.[1] Of course the data for Africa, South America, and Asia are particularly imperfect, but probably the general import of the figures is essentially true. We may push the essence of Clarke's interpretation a step farther and point to the general correspondence of these figures with the degrees of surface cultivation that obtain in these continents. These stand in essentially the same order, viz., Europe, Asia, North America, South America, and Africa. Of course other factors than soil cultivation enter into the results, but it is obvious that a much cultivated surface, softened by soil-tilth and left naked in its softened state for at least a portion of the year and perhaps for all the year, will suffer much more rapid denudation, other things being equal, than surfaces that are constantly mantled with vegetation and whose soils are knit into coherence by a mat of roots. The distinction between the resistance to denudation of native surfaces in humid areas under temperate and tropical conditions, on the one hand, and well-cultivated surfaces, on the other, seems to find peculiar exemplification in the data of Dole and Stabler—the best now available from which to draw tentative generalizations for working purposes. The measurements on which their results are based have been made in very recent years, in the main, and represent the rate of denudation incident to the present state of surface culture. Soil wastage is now notably high. The raising of corn, tobacco, cotton, and potatoes is peculiarly tributary to the leaching and wash of soils. In a somewhat different way, roads are also specially tributary to wash. In very marked contrast to the present state of the surface was its condition just previous to settlement by the whites. The soil of the forested regions was not only protected by a permanent overgrowth of trees and a tangled undergrowth of bushes and herbaceous plants and by a mat of leaves, twigs, and fallen timber, but by a network of roots and rootlets which bound the soil together. The flow of surface drainage was delayed and equalized by the one group of agencies while the soil was rendered resistant to the relatively gentle water action thus insured by the other. On the prairies, the dense mat of native

[1] F. W. Clarke, "A Preliminary Study of Chemical Denudation," *Smith Misc. Coll.*, LVI, No. 5, (1910), p. 7.

grass was abetted by an even denser mat of root fibers to which it adhered tenaciously at its base, and these together made the native sod peculiarly resistant to erosion. In the ravines and meadows the rank "slew" grass, attached even more tenaciously to a tough mass of fibrous roots, was an especially effective defense against the action of floods and freshets where they were liable to do their most effective work. In those early days, as I distinctly remember, the ravines and upland valleys in times of freshets usually ran clear save that their waters were amber-tinged from vegetal extract. The banks of the brooks were then not only close-sodded to the very water's edge, but by their gradual growth closed in on the narrow, pellucid stream between them. The same brooks now, under close pasturage and the feebler turfing of the exotic grasses that have replaced the native sod, have cut open ditches, several times as wide and these are being further widened anually. Under the native conditions even the floods of spring time were but slightly turbid, whereas now the flush of every shower runs black with sediment. Comparing earlier and later impressions of identical areas in the Mississippi Valley, where cultivation has followed native conditions, whether of forest, plain, or meadow, the rate of denudation under culture seems clearly to be some appreciable multiple of the earlier rate, perhaps a very notable multiple. The matter should be determined by direct trials where either the conditions are under complete control or the data for comparing areas not under control are complete and well in hand.

The statistics of Dole and Stabler[1] give for the surface denudation of the United States taken as a whole a mean rate of 1 foot in 9,120 years. For our purpose it is important to know whether this and other large averages are suited for use in the study of circum-continental shelves when these are dependent for their growth chiefly on sediment brought from the drainage slopes of the sea borders. It might seem a natural inference that the coastal slopes should receive a larger rainfall, in general, than the average surface of the continent and so perhaps be denuded more rapidly. But rainfall commonly increases the vegetal clothing and this tends

[1] *Water Supply and Irrigation Papers 231–236*, "Denudation," p. 83.

the other way. To see whether the rates of denudation of the coastal border tracts are essentially the same or essentially different from those of the general continental surfaces, the rates given by Dole and Stabler for the districts of the North Atlantic (1 foot in 13,200 years), the South Atlantic (1 foot in 8,520 years), the North Pacific (1 foot in 19,200 years), and the South Pacific (1 foot in 9,320 years), including in each case only the area within the United States, were combined and compared. The rates, averaged without weighting, were found to give a common rate of 1 foot in 12,180 years. If the Laurentian basin (1 foot in 19,320 years) is reckoned in, the unweighted mean rate rises to 1 foot in 13,476 years. If the Colorado River basin (1 foot in 5,280 years) is also reckoned in, the mean falls back to 1 foot in 12,110 years.

The data for some of the sub-districts are very suggestive. For example, the basins of the Penobscot (1 foot in 24,000 years), the Kennebec (1 foot in 25,200 years); the Androscoggin (1 foot in 21,600 years), the Presumpscot (1 foot in 25,200 years), the Saco (1 foot in 25,200 years), the Merrimac (1 foot in 32,400 years), and the Connecticut (1 foot in 24,000 years) are averaged without weighting, the mean rate is 1 foot in 25,371 years. These notably low rates are probably due in part to the numerous catchment basins on the morainic surfaces of these New England basins, but in no small part also are they probably due to the fact that large portions of New England once under plow have been permitted in recent years to return to a wooded or grassy state. It is of interest to note here that the drainage area of Lake Superior, so far as it lies in the United States, which has only been brought partially under culture, has a denudation rate of 1 foot in 37,200 years.

The erosion of the South Atlantic district (1 foot in 8,520 years) is $2\frac{3}{8}$ times as fast as that of the North Atlantic district (1 foot in 13,200 years), though the gradient of the former is below rather than above that of the latter. Of like import is the erosion rate of the North Pacific slope (1 foot in 19,200 years) when compared with that of the South Pacific slope (1 foot in 9,320 years) more than twice as fast, though the rainfall of the former is much the higher and its gradients certainly not less steep. One of the most vital factors, probably the most vital factor, in these strong

contrasts is with little doubt the different degrees of protection afforded by their vegetal mantles.

It appears then that a review of the best data that we have relative to the present rate of denudation of the coastal slopes gives a mean rate of about 1 foot in 12,000 years. It is to be noted that this embraces tracts that reach to the border of the 30° belt where humidity is low, as well as tracts in the fairly humid mid-latitudes, and that the denudation rate of the latter is scarcely a half of that of the former in spite of the theoretical presumption that denudation should rise and fall with the precipitation. The fact that the efficiency of the vegetal mantle as a protection against erosion also rises and falls with precipitation seems to much more than offset the direct effects of increased drainage flow.

Returning to the broader question, it appears that the best available data relative to the rate of denudation on American coastal slopes at present gives a mean rate of 1 foot in 12,000 years. This is but half the higher rate usually employed in the past, based on the rate for the Mississippi Valley as a whole, 1 foot in 6,000 years. The rate 1 foot in 12,000 years still needs to be corrected for (1) the effect of the present conditions in accelerating denudation, (2) the portions of the sediments lost to the abysmal basins, (3) the portions of the solutions that are held permanently in the oceans as part of the saline element of sea water, and (4) the portions of the solutions that are precipitated to the ocean depths as the hard parts of pelagic animals and plants.

But to curtail this discussion, let us leave these corrections in abeyance and proceed on the basis of the present denudation rate, keeping in mind that it must be corrected to give true results.

Recalling that we had previously fixed upon a belt 200 miles in width as representative of the area directly tributary to the shelf, upon 100,000 miles as its length, and upon 120 miles as the mean width of the shelf, and using the denudation rate 1 foot in 12,000 years, without correction, the formation of the post-Proterozoic terrace would take approximately 108,000,000 years. This is of about the same order of magnitude as the periods usually reached for post-Proterozoic time by employing data based on general denudation in which the present rate is taken as the secular

rate. While our results must be corrected for the difference between present rates of denudation and the mean secular rate—and this correction is quite certainly large, involving other factors than the cultural one—comparisons with other modes of attack must be made previous to such corrections if these have not similarly entered into the results of these other studies.

This general concurrence in results seems to imply that the continents have made peripheral growths of the same order as their other forms of growth. This in turn implies that the continental borders have had a permanency and stability of the same order as the continents themselves.

PETROLOGICAL ABSTRACTS AND REVIEWS

EDITED BY ALBERT JOHANNSEN

McLINTOCK, W. F. P. *Guide to the Collection of Gemstones in the Museum of Practical Geology.* London, 1912. Pp. 92.

This pamphlet is much more than a guidebook, the first thirty-four pages being devoted to a general discussion of the properties of gemstones, the manner of cutting, etc. The remaining pages are devoted to descriptions of the different gems found in the collection, information being given in regard to crystal form, occurrence, methods of cutting, coloring, etc. American museums might well follow the plan of those of Scotland and England in issuing these useful guides. A. J.

MEAD, W. J. "Some Geological Short-Cuts," *Econ. Geol.*, VII (1912), 136–44.

The author gives some ingenious methods for shortening the labor of various computations. The first of these is a graphical method for the determination of the composition of a rock from its mineral constituents. It consists in reading directly from diagrams the component oxides of the minerals. Unfortunately for the majority of petrographers, the author reproduces only one table from a set of fifty-four of rock-forming and thirty-seven of ore minerals, prepared in blue print form for his students at the University of Wisconsin. For the same purpose, but in much more convenient form, is the author's "geologist's slide rule." This consists of a circular diagram, based on logarithmic principles. The percentages are read by means of an inner rotating disk and a celluloid arm. In the present slide rule, thirty-six common rock-forming minerals are included, and the author has in preparation a similar one for thirty-eight of the principal ore minerals. A third short-cut is the "straight-line method" by which direct comparison may be made, without recalculation, of analyses of fresh and altered rocks to show the change in composition due to alteration. A. J.

MENNELL, F. P. *A Manual of Petrology.* Chapman & Hall, London, 1913. Pp. 256, figs. 124. 7*s.* 6*d.* net.

This work, originally planned as the third edition of Mennell's *Introduction to Petrology*, was so much changed during revision, that a

new title was used. The book is designed for the working geologist while "the more elementary portions are chiefly intended for the student"! As has been the case with many recently issued petrological textbooks, the first portion is given up to the rock-forming minerals, 80 pages being here so employed. Chapters are devoted to the classification, structure, origin, metamorphism, and weathering of rocks, and various rock types are discussed. The author bases his classification of the igneous rocks upon their chemical composition, and divides them, in a very simple manner, by their silica content, thus:

	Plutonic	Intrusive	Effusive
Acid, over 65 per cent silica	Granite	Granophyre	Rhyolite
Sub-acid, 60 to 65 per cent	Syenite	Orthophyre	Trachyte
Sub-basic, 55 to 60 per cent	Diorite	Porphyrite	Andesite
Basic, 45 to 55 per cent	Gabbro	Dolerite	Basalt
Ultrabasic, less than 45 per cent	.....	Picrite	Limburgite

This classification is simple enough, but it does not show genetic relationships. It would be impossible, by the use of the microscope, to classify rocks according to this system. Chemical analyses would have to be made of every rock, a prerequisite entirely out of the question in practical work. Furthermore, the names thus limited by the silica content have already been used for rocks of certain mineralogical composition, and the new definitions for the old terms cause further confusion in an already almost hopelessly confused nomenclature.

ALBERT JOHANNSEN

MERRILL, GEORGE P. "On the Supposed Origin of the Moldavites and Like Sporadic Glasses from Various Sources," *Proc. U.S. National Museum*, XL (1911), 481–86.

The greenish, chrysolite-like glass pebbles found in many regions and called moldavite, billitonite, australite, obsidianite, or obsidian bombs, are all included by Suess under the name tektites, and are regarded by him as being of ultra-terrestrial origin, the markings being a consequence of their mode of origin. To this Merrill takes exception, regarding the Bohemian and Moravian specimens as water-worn pebbles of weathered glass, originally etched by corroding vapors or solutions, and the Australian forms as pebbles, water-worn or abraded by wind-blown sands. He does not wish to controvert the theory of an original cosmic

origin, but thinks until they have been seen to fall, that their source is to be found only in the conditions under which they occur.

A. J.

MERWIN, H. E. "A Method of Determining the Density of Minerals by Means of Rohrbach's Solution Having a Standard Refractive Index," *Amer. Jour. Sci.*, XXXII (1911), 425–28.

After making specific-gravity determinations by means of Rohrbach's heavy solution, the writer determines the density of the latter by means of its refractive index. A comparative table of values of density at 20° C. and corresponding refractive indices is given.

A. J.

MERWIN, H. E. "Quartz and Fluorite as Standards of Density and Refractive Index," *Amer. Jour. Sci.*, XXXII (1911), 429–32.

Quartz and fluorite from different parts of the world were compared and it was found that at ordinary room temperature the density of pure quartz is $2.6495 \pm .0010$ and of pure fluorite $3.180 \pm .001$. By sodium light the refractive index (ϵ) of quartz was found to be $1.5443 - 1.5442$, and of fluorite, $1.4338+$ and $1.4338-$.

A. J.

MERWIN, H. E. "Media of High Refraction for Refractive Index Determinations with the Microscope; Also a Set of Permanent Standard Media of Lower Refraction," *Jour. Wash. Acad. Sci.*, III (1913), 35–40.

To fill the gap between fluids having refractive indices from 1.33 to 1.80 and from 2.1 to 2.4, Merwin proposes solutions of iodoform, tri-iodide of arsenic, tri-iodide of antimony, tetra-iodide of tin, and sulphur in methylene iodide. With various proportions dissolved in 100 parts of methylene iodide, fluids of refractive indices between 1.764 and 1.868 are obtained. Fluids from 1.74 to 2.28 were obtained by dissolving arsenic trisulphide in methylene iodide near its boiling-point. Merwin also prepared resin-like substances with indices between 1.68 and 2.10 by dissolving tri-iodides of arsenic and antimony in piperine. For media between 2.1 and 2.6, he used mixtures of amorphous sulphur and arsenic trisulphide. Other media were mixtures of piperine and rosin for indices between 1.546 and 1.682, and mixtures of rosin and camphor for 1.510 to 1.546.

A. J.

MERWIN and LARSEN. "Mixtures of Amorphous Sulphur and Selenium as Immersion Media for the Determination of High Refractive Indices with the Microscope," *Amer. Jour. Sci.*, XXXIV (1912), 42–47.

For minerals having very high refractive indices, Merwin and Larsen propose using molten sulphur, molten selenium, and mixtures of the two, these substances being miscible in all proportions when in a molten condition. The mixtures are prepared by placing the required weight of powdered selenium in a three-inch test tube, heating it until the mineral is thoroughly fused, and allowing it to cool. The proper amount of pure flowers of sulphur is now added and the mixture heated just enough to allow thorough mixing with a glass rod. As the material cools it is gathered on the rod and is cut into small fragments. These may now be returned to the tube, which should be corked, and preserved for use.

To determine refractive indices with this preparation, a small piece of it and a little of the mineral, finely pulverized, are heated together on an object-glass and under a cover-glass, over a small flame, until the preparation is liquid, when the two are mixed and pressed into a thin film. The film is again heated for half a minute until bubbles begin to appear, when it is again pressed thin and cooled, after which the determination is made in the usual manner.

A. J.

MILCH, L. "Grundzüge der Kristallographie," *Taschenbuch f. Math. u. Phys.*, Leipzig u. Berlin, 1913, 359–81.

MILCH, L., and RENZ, CARL, "Ueber griechische Quarzkeratophyre," *Neues Jahrb.*, XXXI (1911), 496–534.

In Argolis and on Hydra, quartz keratophyres and keratophyre tuffs appear, probably of Devonian age. Seven analyses are given.

A. J.

MILCH, L. "Ueber die Beziehungen des Riesengebirgsgranits, (Granitit) zu dem ihn im Süden begleitenden Granitzuge," *Centralbl. f. Min., etc.*, 1911, 197–205.

In an earlier paper Milch had asserted that the two granite areas in question form a single intrusion, the two-mica granite being simply an altered phase of the granitite. Rimann maintained that the two areas represented distinct intrusions. The present paper is a reply to Rimann, and reasons are given for the contention that the mass is a unit.

A. J.

MILCH, L. "Die Systematik der Eruptivgesteine. Erster Teil," *Fortschr. d. Min. Kristallog. u. Petrog.*, III (1913), 189–227.

A history of the development of mineralogical classifications of rocks. Chemical classifications are to be treated in the volume for 1914.

A. J.

MILLER, BENJAMIN L. "The Geology of the Graphite Deposits of Pennsylvania," *Econ. Geol.*, VII (1912), 762–77.

MILLER, WILLIAM J. "The Garnet Deposits of Warren County, New York," *Econ. Geol.*, VII (1912), 493–501.

The author believes the garnets to be due to the assimilation of the Grenville sediments by the intruded igneous rocks.

A. J.

MILLER, WILLIAM I. "Contact Action of Gabbro on Granite in Warren County, New York," *Science*, XXXVI (1912), 490–92.

The author distinguishes nine zones, passing from the typical gabbro to the typical granite country rock. He gives the mineralogical composition of each zone.

A. J.

MOORE, ELWOOD S. "Hydrothermal Alteration of Granite and the Source of Vein-Quartz at the St. Anthony Mine," *Econ. Geol.*, VII (1912), 751–61.

PAIGE, SIDNEY. "The Geologic and Structural Relations at Santa Rita (Chino), New Mexico," *Econ. Geol.*, VII (1912), 547–59.

PAIGE, SIDNEY. "The Origin of Turquoise in the Burro Mountains, New Mexico," *Econ. Geol.* VII (1912), 382–92.

The author gives arguments both for the deep-seated origin of turquoise and for its origin in the zone of weathering. He apparently inclines to the latter belief.

A. J.

RAMSAY, WILHELM. "Beiträge zur Geologie der Halbinsel Kanin," *Fennia*, XXXI (1911), No. 4, pp. 45.

A geologic history of the Kanin Peninsula. An analysis of diabase is given.

A. J.

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—Bagg, R. M. Pliocene and Pleistocene Foraminifera from Southern California. [U.S. Geological Survey Bulletin 513. Washington, 1912.]

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